"Event-Ready-Detectors" Bell Experiment via Entanglement Swapping

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Using independent sources one can realize an "event-ready" Bell–Einstein-Podolsky-Rosen experiment in which one can measure directly the probabilities of the various outcomes including nondetection of both particles. Our proposal involves two parametric down-converters. Subcoherence-time monitoring of the idlers provides a noninteractive quantum measurement entangling and preselecting the independent signals without touching them. We give the conditions for high fringe visibility and particle collection efficiency as required for a Bell test.

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Entanglement, which is at the root of Bell's theorem, usually is considered to be a consequence of some interaction of the particles in their common past. In a seminal paper Yurke and Stoler [1] have proposed that entanglement may arise in the coincidence count rates of particles originating from independent sources. We will show that such a scheme requires precise statements, beyond immediate intuitive expectation, on the coincidence time windows, and that Bell–Einstein-Podolsky-Rosen (EPR) phenomena only occur if the emission acts of the independent sources are monitored with time resolution significantly sharper than the coherence time of the radiation fed into the interferometric setup. If that condition is met one can entangle particles which do not even share any common past. This technique, which we call *entanglement swapping*, leads to a realizable scheme of "event-ready detectors." In such a scheme, which has been called for by Bell since 1971 [2], one knows, via some initiating event, when a pair has been produced. Consequently, one can measure directly the probabilities of the various outcomes, including even nondetection of the particles [2–4] and thereby directly test Bell's inequality. In existing Bell experiments only relative probabilities were accessible. Previous event-ready detectors [4] were interacting directly with the particles and this leads to disentanglement [5].

Consider Fig. 1. Two independent sources emit one pair of entangled photons each. A simplified [6] representation of the resulting four-photon state is the product of $\sqrt{1/2}(|a\rangle|b\rangle+|a'\rangle|b'\rangle)$ with $\sqrt{1/2}(|c\rangle|d\rangle+|c'\rangle|d'\rangle)$. The photons in the beams a,a' and b,b' are not entangled with photons in the beams c,c' and d,d'. We call the two photons in a,a' and d,d' signals and the other two idlers. Suppose we register an idler in detector i_1 in coincidence with an idler in detector i_2. Then, the state of the signals collapses into the *entangled state* $\sqrt{1/2}(|a\rangle|d'\rangle+|a'\rangle|d\rangle)$. This state implies correlations violating Bell's inequality in an experimental setup shown by the dashed lines in Fig. 1. This entanglement between the signals is a consequence of both the initial signal-idler entanglements from each source and the fact that coincident registration in detectors i_1 and i_2 projects the idler photons into the state $2^{-1/2}(|b\rangle|c'\rangle+|b'\rangle|c\rangle)$. The resulting entanglement swapping is a noninteractive quantum measurement of the signals, without touching them, via interacting with the idlers. The experiment can be arranged such that all registration events occur outside each other's light cones. We mention in passing that registration of the idlers in other detectors can collapse the signals into entangled states orthogonal to the one discussed above. The now entangled signal particles do not share any common past.

We assume in our discussion that whenever a pair of idlers are registered in coincidence at i_1 and i_2, each source contributed only one particle. Of course it is equally likely that both particles came from the same source. Fortunately in the laboratory these events can be

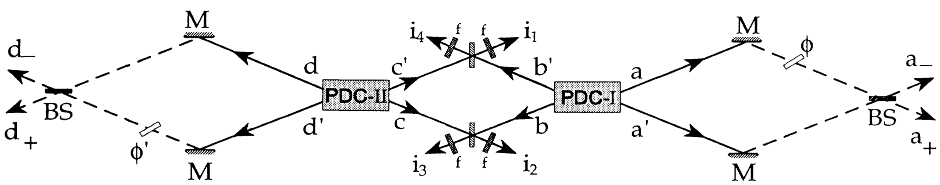

FIG. 1. Principle of an event ready Bell-EPR experiment. Two down-conversion sources PDC-I and PDC-II emit a photon pair each. The specific geometry of each PDC source is obtainable by a suitable arrangement of mirrors and apertures. The initially independent signal photons get entangled by coincident registration of the idlers. (M mirrors; BS beam splitters; f filters; ϕ,ϕ' local phase shifters.)

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distinguished and excluded because they lead to coincident signal counts at the same end of Fig. 1. Although our experiment depends on accidental coincidences between independent sources, we assume the count rates to be low enough such that we are not troubled by triple accidental emissions. Also, since we are interested in those traits of the experiment distinguishing it from the standard ones, we assume for most of our discussion perfect detectors.

The meaning of coincident detection in such an experiment and its implications for Bell's theorem require careful analysis. Note that even for ideal devices with perfect time resolution one always has to impose a finite time gate to *define* two counts as coincident. Furthermore, the bandwidth $\Delta\omega$ implies that the detection time of a signal is determined by the registration time of its idler up to around its coherence time $T_c \approx 1/\Delta\omega$ and *vice versa* [7]. Since down-conversion radiation is extremely broadband, one defines in a practical experiment the bandwidth through apertures and filters. Because of the phase matching condition for frequency (essentially, energy conservation), the filtering of, say, the idlers also limits the bandwidth of the coincident signals. Thus we consider idler filtering only.

Consider first (*experiment A*) the immediate intuitive choice to accept as coincident two idlers arriving at the detectors i_1 and i_2 within a time window $\tau_i = T_c$. This implies that two signals are within the setup and thus we "activate" the signal detectors (this could also be done by associating a certain pair of signal detections with a given pair of registered idlers via an analysis of the arrival times of all photons, long after the actual events). Our detectors will sooner or later register both signals. Now, a signal caught earlier can be thought with a higher probability to be paired with the idler registered earlier. Therefore, the larger both the time separation between the registrations of the idlers and that between the registrations of the signals, the more signal path information we have. The resulting *partial distinguishability* of the paths taken by the signals leads to a *reduction of the two-particle interference contrast.* This reduction keeps the experiment from violating Bell's inequality if the visibility is below 71% [4].

A possible improvement (*experiment B*) would be to only accept *signal* detection events in *subcoherence-time coincidence* (a time window sufficiently smaller than the coherence time). Then the registration times of the idlers, which may still differ by up to T_c, do not provide information about the paths taken by the signals. This implies a very high contrast of the signal fringes. However, such a procedure of postselecting pairs beyond the analyzers only retains a small subset of the full ensemble. Within the full ensemble, Bell's inequality will not be violated because of the large number of rejected pairs. And within the subset, a testable inequality cannot be derived without some auxiliary assumption.

To overcome the problems of experiments A and B let us impose subcoherence-time coincidence (*experiment C*) solely on the registration of the idlers. This implies high visibility: the idlers are registered within such a narrow time window that we have no signal path information. The signal coincidence window could substantially exceed T_c, and it actually should in order to *register almost all signal pairs*, associated with these subcoherence-time idlers. Thus this experiment using a *preselection* procedure can test Bell's inequality, with no problems besides the imperfections of the setup. The ensemble of particles (signal detection events) of the Bell experiment is defined now by the subcoherence-time coincident detections of the idlers, prior to any interaction of the signals with the phase shifters, beam splitters, and detectors. This procedure is a realization of Bell's dream of an experiment with event-ready detectors [2,4]: Our subcoherence-time coincident registration of the idlers "activates" the signal detectors.

We now turn to a quantitative analysis of the experiments A, B, and C. The two-photon state produced by PDC-I, which we consider to incorporate also the idler filters, can be described as [8]

$$\sqrt{1/2}\int d\omega_s \int d\omega_i\, \Delta(\omega_s+\omega_i-\omega_p) f(\omega_i;\omega_f,\Delta\omega)(|\omega_s,a\rangle|\omega_i,b\rangle + |\omega_s,a'\rangle|\omega_i,b'\rangle)\,, \quad (1)$$

where, e.g., $|\omega_s,a\rangle$ ($|\omega_i,b\rangle$) describes the signal (idler) of frequency ω_s (ω_i) in beam a (b), and ω_p is the pump frequency. The function $\Delta(\omega_s+\omega_i-\omega_p)$ reflects the phase matching condition, and in practice can be replaced by Dirac's delta function. By $f(\omega_i;\omega_f,\Delta\omega)$ we denote the transmission function of a filter of central frequency ω_f. A similar state describes the emission by source II.

The detection of an idler in i_1 at t_1 and of another one in i_2, at t_2, causes a wave packet collapse into the *entangled state of the signals*

$$|\Psi\rangle = \sqrt{1/2}(|t_2,a\rangle|t_1,d'\rangle + |t_1,a'\rangle|t_2,d\rangle)\,.$$

The ket $|t_1,a\rangle$ is $\int d\omega\, A(\omega,t_1)h(\omega)|\omega,a\rangle$, where

$$A(\omega,t_1) = \exp[-i(\omega_p-\omega)t_1]$$

describes the amplitude to find a photon (here an idler) with frequency $\omega_p-\omega$ at time t_1 and the function $h(\omega) = Nf(\omega_p-\omega;\omega_f,\Delta\omega)$, N is a normalization constant. The surprising feature of $|\Psi\rangle$ is that, as we shall see below, despite being maximally entangled it leads to the Bell theorem only provided $|t_1-t_2|$ is sufficiently smaller than T_c.

Subsequently at some instant t we register a signal emitted by PDC-I. We check whether the signal emitted by PDC-II and detected at t' satisfies $|t-t'| < \tau_s$. The quantum prediction for the probability $p(i,j|\phi,\phi';\tau_s,\tau_i)$ of having counts both in beam a_i and d_j ($i,j=\pm$) within τ_s, *provided we have detected two idlers* in i_1 and i_2

within $|t_1-t_2|<\tau_i$, is given by

$$\frac{1}{2\tau_i}\int_{-\tau_i}^{\tau_i} d\tau' \int_{-\tau_s}^{\tau_s} d\tau \int_{-\infty}^{\infty} dt\, |2^{-3/2}[H(t'-t_1)H(t-t_2)+ije^{-i(\phi+\phi')}H(t-t_1)H(t'-t_2)]|^2 , \tag{2}$$

where $\tau=t'-t$, $\tau'=t_1-t_2$, the symbol ij equals $+$ for $i=j$ and $-$ otherwise, and $H(t)$ denotes the normalized Fourier transform of $h(\omega)$. Let $P(\tau_s|\tau_i)$ be the conditional probability of registering the event-ready prepared signal pair at any two detectors within τ_s, provided both idlers were detected within τ_i, and $V(\tau_s,\tau_i)$ be the signals fringes visibility under these conditions. Then

$$V(\tau_s,\tau_i)P(\tau_s|\tau_i) \le \frac{1}{2\tau_i}\left|\int_{-\tau_i}^{\tau_i} d\tau' \int_{-\tau_s}^{\tau_s} d\tau \int_{-\infty}^{\infty} dt\, H(t+\tau)H(t+\tau')H^*(t)H^*(t+\tau+\tau')\right| , \tag{3}$$

where the equality holds if the phase of $H(t)$ is a linear function of t.

Let us introduce for $f(\omega_i;\omega_f,\Delta\omega)$ an approximated Fabry-Pérot function $(\omega_i-\omega_f+i\Delta\omega/2)^{-1}$ [9]. The Fourier integrals of the spectral functions $h(\omega)$ can then be approximated by extending their range to negative frequencies. The Fourier transform $H(t)$ reads $(\Delta\omega)^{1/2}\times\exp[-\Delta\omega t/2-i(\omega_p-\omega_f)t]\Theta(t)$, where $\Theta(t)$ is the step function. Thus, our specific choice of f (and hence h) ensures that this approximation does not imply a loss of causality in the description of the filters. The coincidence gates can be expressed in the natural units $\mathcal{T}_i=\tau_i/T_c$ and $\mathcal{T}_s=\tau_s/T_c$. For finite τ_s the probability $P(\tau_s|\tau_i)$ then reads $1-\sinh\mathcal{T}_i\exp(-\mathcal{T}_s)/\mathcal{T}_i$ for $\tau_i<\tau_s$, and it is $[\mathcal{T}_s-\exp(-\mathcal{T}_i)\sinh\mathcal{T}_s]/\mathcal{T}_i$ for $\tau_i>\tau_s$. In general one has

$$V(\tau_s,\tau_i)P(\tau_s|\tau_i)=\frac{[1-\exp(-\mathcal{T}_i)][1-\exp(-\mathcal{T}_s)]}{\mathcal{T}_i} . \tag{4}$$

The actual form of the expressions is a consequence of the specific f chosen. Nevertheless, one can show that any sensible filter function gives qualitatively similar results: The expression can have high numerical value (above 0.71) only provided $\mathcal{T}_i$ is sufficiently smaller than 1 (i.e., for ultracoincident idlers), and $\mathcal{T}_s$ much bigger than 1.

Let us now consider Bell's inequality in the form [2]

$$E_{\rm hv}(\phi_1,\phi_1')+E_{\rm hv}(\phi_2,\phi_2')+E_{\rm hv}(\phi_2,\phi_1')-E_{\rm hv}(\phi_1,\phi_2')\le 2 , \tag{5}$$

where the correlation function $E_{\rm hv}(\phi,\phi')$ is defined as $\sum_{ij} ij\, p_{\rm hv}(i,j|\phi,\phi';\tau_s,\tau_i)$, and $p_{\rm hv}(i,j|\phi,\phi';\tau_s,\tau_i)$ in turn is any local realistic prediction for the probability that the detectors a_i and d_j fire (under the conditions specified by τ_s and τ_i). The definition of $E_{\rm hv}$ must include the fact that some signals may not fit into the required time gate τ_s. Following [2,4] we let the result be $ij=\pm 1$ if the signals were detected within τ_s, or 0 if a signal photon from an event ready pair arrived too late.

The quantum prediction for the correlation function for the specific time gates reads $V(\tau_s,\tau_i)P(\tau_s|\tau_i)\cos(\phi+\phi')$. Thus, the product $V(\tau_s,\tau_i)P(\tau_s|\tau_i)$ gives the modulation amplitude of the quantum correlation function. Bell's inequality (4) is only violated if

$$V(\tau_s,\tau_i)P(\tau_s|\tau_i)>\sqrt{1/2}\approx 0.707 . \tag{6}$$

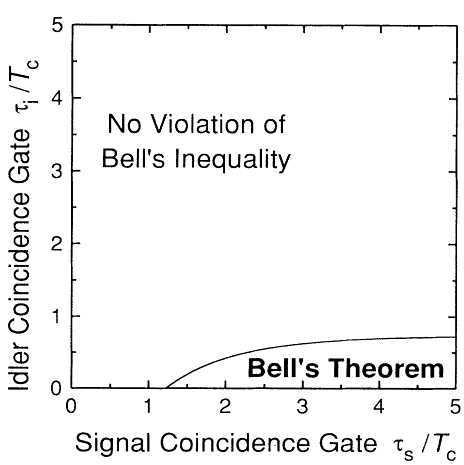

FIG. 2. Boundary of the experimental parameters (6) necessary to violate Bell's inequality (for our model of the filter).

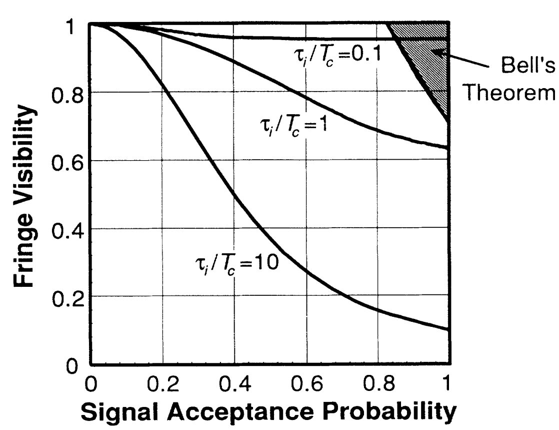

FIG. 3. Two-photon interference visibility versus signal acceptance probability, which is defined as the square root of the probability $P(\tau_s|\tau_i)$ to register both signals within τ_s given both idlers were registered within τ_i. The three curves are obtained by sweeping τ_s from 0 to ∞ with fixed τ_i. Note that ultracoincident idlers are necessary to violate Bell's inequality.

In Fig. 2 we show the region of time gates implied by (4) and (6). Violation of Bell's inequality occurs only if both (a) the idlers are detected in subcoherence-time coincidence and (b) the signal coincidence window is significantly longer than T_c. The specific details of the interdependence of the signal fringe visibility $V(\tau_s, \tau_i)$ and the signal acceptance probability, defined as $[P(\tau_s|\tau_i)]^{1/2}$, can be seen in Fig. 3.

Like all experiments testing local realism performed so far, the ones proposed here also would suffer from a finite detector efficiency and an abundance of instrumental imperfections which lower the visibility. Of course, if the detector efficiency is not sufficiently high, one must resort to some auxiliary assumption such as fair sampling, as in the past. Nevertheless, if there are some limitations intrinsic to the setup, we must work in a very sharp subcoherence-time regime ($\tau_i \ll T_c$).

Finally, it has not escaped our attention that using a similar technique one can obtain a source exhibiting Greenberger, Horne, and Zeilinger (GHZ) correlations. We also note that subcoherence-time coincidence is a general requirement in experiments involving Bell measurements of particles from different sources, e.g., in quantum "teleportation" [10].

In conclusion, we remark that such an experiment with event-ready registration of independent photons might be a further step toward a definitive test against local realism.

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