1.1 Gaussian and SI units (version 1.2)

The two most widely used systems of units are the Gaussian system which uses centimeters, grams, and seconds for the fundamental quantities of length, mass, and time, and SI (Système International) which uses meters, kilograms and seconds for these quantities. When converting from Gaussian to SI units numerical values are related by the scaling factors

$$x_G = k_x x_{SI}, \quad m_G = k_m m_{SI}, \quad t_G = t_{SI}$$

where $k_x = 100$ and $k_m = 1000$. Consider, for example, energy which has units of $x^2 s^{-2} m$. If the numerical value is U_G in the Gaussian system then in SI units the energy is related by

$$U_G \sim x_G^2 m_G \rightarrow (k_x^2 k_m) x_{SI}^2 m_{SI} \sim (k_x^2 k_m) U_{SI}.$$

Thus

$$U_{SI} = \frac{1}{k_x^2 k_m} U_G = 10^{-7} U_G$$

which agrees with the known relation 1 erg $= 10^{-7}$ J.

While it is easy to transform quantities such as energy between the systems, when dealing with electromagnetism confusion can arise since the same physical quantities have different units as well as different numerical values in the two systems.[1] The Maxwell equations, constitutive relations between fields and polarizations, and Lorentz force per charge q in Gaussian units are

$$\begin{aligned}
\nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J}, \\
\nabla \cdot \mathbf{D} &= 4\pi\rho, \qquad \nabla \cdot \mathbf{B} = 0, \\
\mathbf{D} &= \mathbf{E} + 4\pi \mathbf{P}, \\
\mathbf{H} &= \mathbf{B} - 4\pi \mathbf{M}, \\
\mathbf{F} &= q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right).
\end{aligned}$$

In the SI system the equations are

$$\begin{aligned}
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}, \\
\nabla \cdot \mathbf{D} &= \rho, \qquad \nabla \cdot \mathbf{B} = 0, \\
\mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P}, \\
\mathbf{H} &= \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}, \\
\mathbf{F} &= q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right).
\end{aligned}$$

In SI the constants ϵ_0, μ_0 have exact values

$$\epsilon_0 \mu_0 = \frac{1}{c^2}, \quad \mu_0 = 4\pi \times 10^{-7}, \quad c = 299792458 \text{ m/s}.$$

[1] This discussion follows the treatment in the book by Schwinger, et al.

To convert any electromagnetic equation from Gaussian to SI units we multiply the Gaussian quantities by scaling factors to get the corresponding SI quantities:

$$\begin{aligned}
&\mathbf{E}_G \to k_E\mathbf{E}_{SI}, \quad \mathbf{D}_G \to k_D\mathbf{D}_{SI},\\
&\mathbf{B}_G \to k_B\mathbf{B}_{SI}, \quad \mathbf{H}_G \to k_H\mathbf{H}_{SI},\\
&\mathbf{P}_G \to k_P\mathbf{P}_{SI}, \quad \mathbf{M}_G \to k_M\mathbf{M}_{SI},\\
&\rho_G \to k_\rho\rho_{SI}, \qquad \mathbf{J}_G \to k_J\mathbf{J}_{SI}.
\end{aligned}$$

Consistency of the Maxwell equations and the Lorentz force law in the two systems fixes the scaling constants to be

$$\begin{aligned}
&k_E = \sqrt{4\pi\epsilon_0}, \quad k_D = \sqrt{\frac{4\pi}{\epsilon_0}},\\
&k_B = \sqrt{\frac{4\pi}{\mu_0}}, \quad k_H = \sqrt{4\pi\mu_0},\\
&k_\rho = k_J = k_P = \frac{1}{\sqrt{4\pi\epsilon_0}},\\
&k_M = \sqrt{\frac{\mu_0}{4\pi}}.
\end{aligned}$$

For example to convert the formula for the Coulomb potential energy between two electrons from Gaussian to SI units we use

$$U_G = \frac{e_G^2}{r} \to \frac{k_\rho^2 e_{SI}^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e_{SI}^2}{r} = U_{SI}.$$

To find conversions between numerical values we must also scale the units of length and mass. Since energy is proportional to mx^2/t^2 we have

$$\frac{e_G^2}{r_G} = (k_m k_x^2)\frac{k_\rho^2 e_{SI}^2}{r_{SI}} = \frac{10^7}{4\pi\epsilon_0}\frac{e_{SI}^2}{r_{SI}} = c^2\frac{e_{SI}^2}{r_{SI}}.$$

Thus the electron charge in Gaussian units is related to the charge in SI units ($e_{SI} = 1.602 \times 10^{-19}$ C) by

$$e_G = e_{SI} \times c \times \sqrt{r_G/r_{SI}} = e_{SI} \times c \times 10 = 4.803 \times 10^{-10} \text{ esu}$$

where esu stands for electrostatic units.