Wave Motion and Optics

Go over information on web page hexagon.physics.wisc.edu, link at top to Ph 325.

The world around us appears to consist of both particles and waves. Quantum Mechanics tells us that also particles act like waves sometimes.

In this course we will learn about many different aspects of waves, and in particular optical waves.

Most of you own a device that interacts with waves at two very different wavelengths - a cell phone.

Microwave frequency signals

Optical frequency light - display + camera discuss.
2) What is a wave?

An excitation that is periodic in space + time.

\[ u(x,t) \]

Call the wave amplitude \( u(x,t) \).

Generic wave equation

\[
\frac{\partial^2 u}{\partial t^2} + a \frac{\partial^2 u}{\partial x^2} = 0
\]

This describes Electromagnetic waves, waves on a stretched string, water waves, etc.

What is the solution to the wave equation?

There are many, in fact infinitely many possible solutions.

Try \( u = c_1 f_1(x-vt) + c_2 f_2(x+vt) \)

where \( c_1, c_2 \) are constants, \( tv \) is the wave speed, or phase velocity.
The functions $f_1, f_2$ are unknown but let’s plug this form of solution into the wave equation.

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial t} \right) = c_1 f_1'(-v) + c_2 f_2'(v)$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial t} \right) = c_1 v^2 f_1'' + c_2 v^2 f_2''$$

and

$$\frac{\partial^2 u}{\partial x^2} = c_1 f_1'' + c_2 f_2''$$

The wave equation gives:

$$\frac{\partial^2 u}{\partial t^2} + a \frac{\partial^2 u}{\partial x^2} = c_1 v^2 f_1'' + c_2 v^2 f_2'' + a c_1 f_1'' + a c_2 f_2''$$

$$= (v^2 + a)(c_1 f_1'' + c_2 f_2'') = 0.$$ 

For any $f_1(x-vt), f_2(x+vt)$ we have a solution if $v^2 + a = 0$ or $a = -v^2$. $c_1$ and $c_2$ depend on boundary conditions.
So,
\[ \frac{\partial^2 u}{\partial t^2} - \nu^2 \frac{\partial^3 u}{\partial x^3} = 0 \]

If the wave speed is \( v > 0 \), the wave is propagating towards +x. So,
\[ u \sim f(\xi - vt) \]

It is common to write this as
\[ u \sim f(kx - \omega t) \]
where \( k \) is the wave number and \( \omega \) is the angular frequency.

Then,
\[ \frac{\partial^2 u}{\partial x^2} = k^2 f'' \]
\[ \frac{\partial^2 u}{\partial t^2} = \omega^2 f'' \]
and
\[ \omega^2 f'' - \nu^2 k^2 f'' = 0 \implies \omega^2 = \nu^2 k^2 \]

or \[ v = \frac{\omega}{k} \]

More definitions:
\[ \omega = 2\pi f ; \quad \nu = \text{frequency [Hz]} \]
\[ k = \frac{2\pi}{\lambda} ; \quad \lambda = \text{wavelength [m]} \]

So,
\[ \frac{\nu}{k} = \frac{2\pi f}{\nu} = \lambda f \]

or \[ \nu = \lambda f \]
Four important quantities

\[ k \quad \lambda \]

\[ \omega \quad \nu \]

And wave speed \( \nu \)

\[ \nu = \frac{\omega}{k} = \lambda \nu \]

\[ \lambda \quad x = \lambda \]

\[ kx = 2\pi \Rightarrow x = \lambda \]

\[ T = \frac{1}{\nu} \] is the wave period in time

\[ \omega t = 2\pi \Rightarrow \nu t = 1 \Rightarrow T = \frac{1}{\nu}. \]

Although any function \( f(kx - \omega t) \) solves the wave equation, the basic solution is

\[ u = \cos (kx - \omega t) \]

Arbitrary wave profiles can then be synthesized by combining waves of different \( k \).
\[ \mathbf{A} = \sum_k c_k \cos(kx - wt) \]

or \[ \int dk \ c(k) \cos(kx - wt) \]

This is known as a Fourier series or integral representation. We will come back to this in much more detail later on.

For "non-dispersive" waves \( v \) does not depend on \( w \) and the pulse propagates without spreading.

Light is an electromagnetic wave - a solution of the Maxwell equations. Light propagates in vacuum or in some material such as a piece of glass.

What is the speed of light? \( v = \frac{w}{k} = \lambda v \).

This question has a long history.

1676 Ole Roemer in principle estimated the speed from observations of Jupiter's moons. \( \sim 214,000 \text{ km/s} \).
A better measurement was made by Fizeau in 1849, 315,000 km/s.

The BYU lab exercise video gives an example of one way of measuring light.

The Maxwell equations for electromagnetism predict a wave speed of

\[ C = \frac{\text{electromagnetic charge}}{\text{electrostatic charge}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]

\[ \varepsilon_0 = \text{electric permittivity} \]
\[ \mu_0 = \text{magnetic permeability} \]

In 1857 Weber + Kohlrausch measured

\[ \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 310,800 \text{ km/s} \]

In 1861 Maxwell proposed that light is an electromagnetic wave.

The modern definition is that the speed of light is defined to be

\[ C = 299,792,458 \text{ m/s} \]
\( \mu_0 \equiv 4\pi \cdot 10^{-7} \text{ H/m} \), \( H \) is a Henry.

\( \varepsilon_0 \equiv \frac{1}{\mu_0 c^2} = 8.854 \ldots \cdot 10^{-12} \text{ F/m}, \) F is a Farad.

Furthermore, the second is defined in terms of a Cs atom. The ground state of Cs has two levels:

\[
\begin{align*}
6s^2 & \quad F=4 \quad \uparrow \quad \text{the frequency } \nu \text{ is defined to be} \\
F=3 & \quad \downarrow \\
9192631770 & \quad \nu = 9192631770 \text{ Hz}
\end{align*}
\]

Thus the second is

\[
9192631770 \times (\text{the time for one oscillation between the } F=3 \text{ and } F=4 \text{ states of Cs}).
\]

Thus, 1 meter is the distance traveled by light in \( \left( \frac{1}{299792458} \right) \) s.

Light waves are very useful. They carry energy, momentum, angular momentum, information (fiber optics).
Is light really a wave?

Light can also be described in terms of particles, photons:

\[
\text{source} \quad \begin{array}{c} \text{attenuator} \\
\end{array} \quad D \quad \rightarrow \quad \text{detector} \quad \text{discrete clicks} \\
\quad \text{- single photons}
\]

Each photon has energy \( E = h \nu \) and momentum \( \vec{p} = \frac{h}{\lambda} \).

Even when light beam is very weak, so we observe discrete clicks from individual photons, we can observe interference.

\[
\text{source} \quad \begin{array}{c} \text{lens wave} \\
\end{array} \quad \begin{array}{c} \text{2 slits} \\
\end{array} \quad \rightarrow \quad \text{screen} \\
\quad \begin{array}{c} \text{d} \sim \frac{\lambda L}{d} \\
\end{array}
\]

will calculate this more precisely later on.

Even if we attenuate the wave so one particle at a time passes the slits, we still see interference with the same period d. We must conclude that photons interfere.
The experiment has also been done with massive particles
electrons
neutrons
atoms
molecules C60 – Bucky balls.

This is a topic for a "quantum" optics course. We will mostly focus on a wave description.