Sagnac Effect - Rotation sensing widely used for optical gyroscopes

\( (1911, 1913) \)
Harress, Sagnac

time for 1 round trip for light copropagating with rotation is

\[ c t_+ = 2 \pi R + R \Omega t_+ \]

\[ t_+ = \frac{2 \pi R}{c - R \Omega} \quad \text{(increased time)} \]

for light propagating against rotation

\[ c t_- = 2 \pi R - R \Omega t_- \]

\[ t_- = \frac{2 \pi R}{c + R \Omega} \quad \text{(decreased time)} \]

time difference

\[ \Delta t = t_+ - t_- = \frac{2 \pi R}{c} \left( \frac{1}{1 - \frac{R \Omega}{c}} - \frac{1}{1 + \frac{R \Omega}{c}} \right) \]

\[ \approx \frac{2 \pi R}{c} \frac{2 R \Omega}{c} = \frac{2 \pi R^2 \Omega}{c^2} \]
Phase difference:

\[ \Delta \phi = \omega \Delta t = c \kappa \Delta t = \frac{c}{\lambda} \frac{4\pi R^2 \Omega}{c^2} \]

\[ = \frac{8\pi}{\lambda c} (R^2) \Omega = \frac{8\pi}{\lambda c} \cdot A \cdot \Omega \]

This is a general result not only valid for a circle.

By analyzing an \( N \) sided regular polygon, we can show that

\[ \Delta \phi = \frac{8\pi}{\lambda c} \hat{A} \cdot \hat{\Omega} \]

with

\[ \hat{A} = \frac{1}{2} \oint \hat{r} \times d\hat{r} \]

Examples:

\( \lambda = 633 \text{ nm (He-Ne laser)} \)
\( A = 4.5 \text{ cm}^2 \)
\( \Omega = 15 \text{ deg/hr} = 7.3 \times 10^{-9} \text{ rad/s} \)

\[ \Delta \phi = 4.2 \times 10^{-8} \text{ rad} \]

Michelson (1920s) used 5 miles of evacuated sewer pipes to measure rotation of the earth.
It is impractical to build extremely large A.

A solution is to use N loops of optical fiber.

Each loop has length $L = 2\pi R$

effective area, $A = N \cdot \pi R^2$

So, $\Delta \phi = \frac{8\pi}{\lambda c} N \pi R^2 \Omega$

total length is $L = N \cdot L = N \cdot 2\pi R$ so

$\Delta \phi = \left(\frac{8\pi}{\lambda c}\right) \frac{LR}{2} \Omega$

Trade off since fiber absorbs light, power decays as $P = P_0 e^{-\alpha L}$
Telecom fiber has loss as low as 0.2 dB/km.

Loops of a few km length are used to get sensitivity at $10^{-3}$ deg/h.

Sensitivity depends on detection noise, something we will talk about in 2nd half of class.

Output signal

For best sensitivity device can be biased at $\frac{\Delta \eta}{2}$ or

$$\frac{\Pi}{2} = \left( \frac{8 \eta}{\lambda c} \right) \left( \frac{4 R}{2} \right) \Omega_{\Delta \eta/2}$$