Fine Structure of the Hydrogen Atom. IV*

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(Received February 29, 1952)

The fourth paper of this series contains an analysis of measurements made in 1950 on the fine structure of hydrogen and deuterium. After application of numerous experimental and theoretical corrections the following results are obtained. The displacement $2S_1 - 2P_1$ is 1058.27 Mc/sec for hydrogen and 1059.71 Mc/sec for deuterium, each with a limit of error ±1.0 Mc/sec. There seems to be satisfactory agreement between these and the most recently calculated values. The difference of 1.44 Mc/sec between deuterium and hydrogen is compatible with an explanation based on reduced mass effects and the finite extension of the deuterion.

The $2P_1 - 2P_1$ separation for deuterium is 10,972.11 Mc/sec (limit of error ±1.0 Mc/sec). With assumption of a well-justified theoretical formula for the doublet splitting, one may calculate a value for the fine structure constant $\alpha$. The result for $1/\alpha$ = 137.033 (limit of error ±0.006) is in distinct disagreement with the value $1/\alpha$ = 137.0429 ± 0.0009 derived from the hyperfine splitting of the ground state of hydrogen.

The formula used for this was in error due to neglect of corrections of relative order $Z\alpha^2$, and is still in doubt because of reduced mass effects and spatial extent of nuclear magnetism.

The preceding papers of this series have laid a foundation in experiment and theory for analysis of more accurate measurements of the fine structure splittings of hydrogen and deuterium. Results of such an analysis are given in the present paper.

The provisional goal was to measure the fine structure pattern with an accuracy of 1 Mc/sec. Considering that the resonances are more than 100 Mc/sec broad, this required working within 1 percent of the half-width which is not usually possible in radiofrequency spectroscopy. In the course of the measurements, however, it became apparent that the internal consistency of data accumulated in a single run was an order of magnitude better than this, while the external consistency of different runs was about at the 0.5 percent level. A

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**Figure 54.** Percentage of quenching of metastable hydrogen atoms as a function of rf voltage applied to transmission line. The plate separation was roughly 1 cm. Curves are shown for magnetic fields of 229 and 1275 gauss.

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* Work supported jointly by the Signal Corps and ONR.
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73. Quenching of beam by Electrostatic Fields

The calculated effect of motional electric fields on metastable hydrogen atoms in states $\alpha$ and $\beta$ was indicated in Fig. 34. Additional quenching may be produced by application of a static electric field in the rf interaction region, or elsewhere along the beam. Examples of this quenching are given in Fig. 54 for magnetic fields of 229 and 1275 gauss. The electrode geometry was not sufficiently good to warrant an attempt to fit these data accurately to a theoretical formula based on an equation like (42). The ease of quenching, however, seems to vary with magnetic field in the manner expected if the beam were composed mostly of atoms in the $\alpha$-state. According to Fig. 34 this should be the case for a range of magnetic fields extending from perhaps 400 to 1200 gauss. Outside of this range the beam should have an appreciable fraction of atoms in the $\beta$-state. In practice, however, this fraction is reduced by the presence of electric fields in the electron bombarder, detector, and possibly elsewhere along the beam, for the $\beta$-state is more easily quenched than $\alpha$ up to $H = 3400$ gauss. This is especially true in the vicinity of the crossing point of $\beta$ and $f$, which occurs at about 1188 gauss where very weak stray electric fields in the $z$ direction may cause a decay of $\beta$.

The behavior at higher fields is shown in Fig. 55, which was taken at 1450 gauss. This indicates that the beam
is composed of an easily quenched "soft" component formed by atoms in the $\beta$-state, and a "hard" component of atoms in state $\alpha$. In such a case, the fractional quenching may be written as

$$\phi = \phi_\alpha [1 - \exp - (V/V_\alpha)^2] + \phi_\beta [1 - \exp - (V/V_\beta)^2],$$

(245)

where $V$ is the voltage applied to quenching electrodes, $V_\alpha$ and $V_\beta$ are constants characterizing the ease of quenching, and $\phi_\alpha$ and $\phi_\beta$ are the fractions of the two kinds of atoms $\alpha$ and $\beta$ in the beam. In Eq. (245) it is assumed that the beam consists only of two components $\alpha$ and $\beta$ and no attempt is made to consider hyperfine splitting or velocity distribution. For $|V| \ll V_\alpha$, one may write

$$\phi = \phi_\alpha (V/V_\alpha)^2 + \phi_\beta [1 - \exp - (V/V_\beta)^2],$$

(246)

so that when $\phi$ is plotted against $V^2$ in the range

$$V_\beta \ll |V| \ll V_\alpha,$$

a straight line results whose slope is $\phi_\alpha/V_\alpha^2$ and whose intercept on the vertical axis is just $\phi_\beta$. In the case of Fig. 55 this analysis yields Fig. 56 with $\phi_\beta = 20.6$ percent; $\phi_\alpha = 79.4$ percent, and $V_\alpha = 26.2$ volts. From the behavior for $|V| < V_\beta$, one may estimate that $V_\beta \approx 2.88$ volts.

When such measurements are made at various magnetic fields between 850 and 1450 gauss, it is found that the fraction $\phi_\beta$ decreases rapidly as the field is decreased from 1450 gauss, passes through a minimum near the $\beta\gamma$ crossing point 1188 gauss and increases slightly and then falls off sharply as the field is lowered further. Typical results have been $\phi_\beta = 22$ percent at $H = 1450$ gauss, and $\phi_\beta = 3$ percent at $H = 1188$ gauss when the apparatus is in a "clean" condition, but lower figures such as 14 percent and 0.5 percent, respectively, have been observed even though the beam was not excessively poisoned. The variation is probably to be attributed to varying conditions in the electron bombarder of the sort discussed in Sec. 43.

It was discovered that on some occasions there was a rather considerable effect on the beam when the quenching voltage was merely reversed in sign. This was attributed to the existence of a contact-like potential difference between the quenching electrodes which could amount to as much as one volt. The actual figure varied considerably from day to day, and sometimes during a run. Presumably this potential difference was in part due to contact potentials and partly to charges produced on insulating films by illumination with ultraviolet light. To allow for this effect, the portion of Eq. (246) linear in $V^2$ was rewritten as

$$\phi = (\phi_\alpha/V_\alpha^2) (V - V_\beta)^2 + \phi_\beta,$$

(247)

with $V_\alpha$ the potential difference with no applied electric field. A procedure was developed whereby the constants $\phi_\beta$, $V_\beta$, and $V_\alpha$ could be calculated rapidly for magnetic fields of interest from measurements of $\phi$ at three voltages in the range $V_\beta \ll V \ll V_\alpha$.

The presence of soft component $\beta$ requires a correction to the observed rf quenching by a factor $1 - (\phi_\beta/\phi_\alpha) = 1 + \epsilon$. For observations between 400 and 850 gauss the motional quenching keeps the beam sufficiently pure $\alpha$, and no soft component correction is needed. It was, however, necessary for data taken at magnetic fields near the $\beta\gamma$ crossing point. One of the unsatisfactory features of the present work is the possible error introduced in application of the rather variable soft component correction. Of course, the frequency was chosen so that $\phi_\beta$ was small not only at the resonance peaks but also at the two working points. Only a difference in soft component at these two points would lead to a significant differential correction that might be in error. It is planned, however, in future work to treat the problem in a somewhat better fashion.

The above complications impose a very severe limitation on the possible ranges of frequency versus magnetic field plot (Fig. 15) which can be explored accurately. The soft component becomes unmanageably large above 1450 gauss, difficult either to correct for or to quench

Fig. 56. Analysis of data shown in Fig. 55 whereby quenching is plotted against square of voltage. The beam is found to contain 20.6 percent of atoms in the more easily quenched lower metastable state.
differentially. The best choice in perhaps \( H \sim 1158 \) gauss, i.e., the second \( \beta \)-crossing. The first \( \beta \)-crossing \( H \sim 575 \) gauss would also be satisfactory except that for \( \alpha \alpha \) the sharp resonances \( \alpha \beta \) (Sec. 47) are superimposed, while for all transitions there is a considerable overlap from other transitions. It is somewhat better to have the resonances occur at a higher field, such as 700 gauss, in order to obtain more separation from nearby peaks, but not yet to have appreciable soft component contamination in the beam. These criteria have guided the choice of transitions for accurate study.

### 74. Forbidden Component

The possibility of transitions with \( \Delta m_I \neq 0 \) has been previously ignored. In one case, namely \( H(\alpha \beta) \), such a transition can occur with appreciable intensity in the present apparatus. Referring to Fig. 17, this is the one from state \( \alpha, m_I = -\frac{3}{2} \) to state \( \beta, m_I = -\frac{1}{2} \), which can be regarded as produced by matrix elements of the hyperfine interaction energy \( \Delta \omega I \cdot J \) which are off diagonal in \( m_I \) and \( m_J \). These mix in with the state \( \alpha, m_I = -\frac{1}{2} \) some contamination of state \( \beta, m_I = +\frac{1}{2} \) which is coupled by electric dipole radiation to \( \beta, m_I = +\frac{1}{2} \). Likewise, some of \( \alpha, m_I = -\frac{1}{2} \) is mixed in with \( \beta, m_I = +\frac{1}{2} \) and makes transitions possible from \( \alpha, m_I = +\frac{1}{2} \).

The relative intensity of transition \( \alpha, m_I = -\frac{1}{2} \rightarrow \beta, m_I = +\frac{1}{2} \) to the allowed transitions \( \alpha, m_I \rightarrow \beta, m_I \) is given by

\[
\text{Ratio} = \frac{\left( \frac{\Delta \omega}{g \mu_B H} \right)^2}{\left( |c| E \cdot r |\alpha| \right)^2} \left( |f| E \cdot r |\alpha| \right)^2
\]

(248)

For \( H(\alpha \beta) \), \( H \sim 704 \) gauss, one has \( \Delta \omega / g \mu_B H \sim 1.6 \). With the particular rf field geometry used, the electric field was primarily in the \( z \) direction and therefore favored the \( \alpha \alpha \) transition over \( \alpha \beta \). From calculations on the amount of overlap of \( \alpha \alpha \) on \( \alpha \beta \), it was estimated that the ratio of squared matrix elements in (248) was 4.74, so that the forbidden component would have a relative intensity of about 4.7 percent. This component occurs at about 12.5 gauss above the center of the \( \alpha \beta \) resonance and contributes a shift of 0.10 gauss to the apparent center of the complex curve. The effect can be neglected for all other transitions observed with the present apparatus.

### P. ANALYSIS OF DATA

#### 75. Nature of Data

The basic plan for determination of the fine structure was outlined in Sec. 45. The following transitions were studied: \( H(\alpha \alpha) \) and \( D(\alpha \alpha) \) at 2195 Mc/sec, \( H(\alpha \beta) \) and \( D(\alpha \beta) \) at 2395 Mc/sec, and \( D(\alpha \alpha) \) at 7195 Mc/sec. The basis for these choices is explained in Sec. 73. It had been hoped to also measure \( H(\alpha \alpha) \) at 7195 Mc/sec as well as \( H(\alpha \beta) \) and \( D(\alpha \beta) \) at 11,800 Mc/sec, but after the possibility of further improvements became apparent, it was decided to postpone the study of these transitions.

The data presented here were obtained in fifteen “runs” which are listed in Table VIII. In general, the apparatus was turned on fully in the morning for “seasoning” and the data were taken during four hours in the afternoon. It required about nine months to accumulate the fifteen runs, since the apparatus had to be opened rather frequently for cleaning as described in Sec. 43.

Data taken for each run consisted basically of about fifteen comparisons of the rf quenching at two points of approximately equal heights on opposite sides of the resonance curve. Each comparison was obtained from the average of six measurements of the fractional quenching on each side. A typical sample of data for one such comparison is given in Table IX from the run of 7/13/50 on transition \( H(\alpha \beta) \).

The values in column \( R_{\text{run}} \) are the nominal resistance box settings on the variable side of the bridge circuit used for determining magnetic field. To these must be added previously determined corrections due to departures from nominal values as well as a measured value of the search coil resistance. After calibration of the permanent magnet by observations of transition \( D(\alpha \beta) \), the magnetic fields corresponding to \( R_n \) and \( R_0 \) could be obtained and were 649.86 and 753.86 gauss, respectively, for this case. Readings of the galvanometer were taken at 20-second intervals, some with \( \text{rf} \) on or off, and some with dc quenching voltage applied to the beam. Columns 1, 3, 5, 7, and 9 correspond to \( \text{rf} \) and dc fields off; columns 2, 4, and 6 to \( \text{rf} \) on and dc off; while column 8 was taken with dc on and \( \text{rf} \) off. The column marked \( \Delta f \) gives the frequency interval below 2400 Mc/sec for the \( \text{rf} \) on readings in the row in question. Since the frequency drift was small it sufficed to record \( \Delta f \) once for each line of data. In order to keep track of \( \text{rf} \) power variations, the crystal current \( I_a \) was recorded in some convenient unit on the intermediate rows in columns 1, 3, and 5. The remaining figures shown under columns, 2, 4, and 6 are the calculated galvanometer deflections due to \( \text{rf} \). Thus, in column 2

\[
(\text{rf quenching})^2 = \frac{1}{2} (G_1 + G_2 - G_3)
\]

(249)

where \( G_n \) is the galvanometer reading recorded in column \( n \). The dc quenchings in column 8 were similarly calculated. Also shown are calculations of average dc and \( \text{rf} \) quenching, average crystal current for each row, and in the final column, the average percentage of \( \text{rf} \) quenching for each line.

Much auxiliary data were required in order to make the calculations necessary to obtain from each com-
Table IX. Transition H(αf), 7/13/50, 2400 − Δf Mc/sec. Sample data showing intercomparisons on two sides of resonance at magnetic fields corresponding to nominal bridge resistances $R_{nom}$. The frequency is 2400 − Δf Mc/sec, subject to corrections applied later because of departure of the standard crystal from 5 Mc/sec. The measurement cycle consists of a reading of dc quenching followed by three determinations of rf quenching, then of dc quenching, then three more rf quenchings, and a final determination of dc quenching. Listed vertically in columns 1, 3, and 5 are galvanometer readings with quenching fields off, and also crystal current readings obtained when the rf is next turned on. In columns 2, 4, and 6 are galvanometer readings with rf on and calculated galvanometer deflections due to rf. In columns 7 and 9 are galvanometer readings and calculated galvanometer deflections due to dc quenching. Also given are the average dc and rf quenchings, average crystal currents, and finally the average percentage of quenching for each line of data.

| $R_{nom}$ | Δf | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | dc   | rf   | %
|-----------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|------|-----
| 4162      |     | 144.3 | 6.0   | 144.3 |     |       |       |       |       |       |      |      |     |
|           | 4.68| 144.3 | 116.3 | 144.5 | 116.3 | 144.7 | 116.5 | 145.2 | 144.8 | 138.7 | 28.28| 20.39|     |
| 4194      |     | 179.5 | 28.1  | 180.0 | 28.3  | 180.5 | 28.45 | 139.0 | 180.0 |       |      |      |     |
| 4.68      |     | 144.8 | 116.1 | 144.7 | 116.1 | 144.6 | 116.2 | 144.4 | 144.3 | 138.68| 28.5 | 20.55|     |
| 4846      |     | 181.5 | 28.65 | 181.0 | 28.55 | 181.5 | 28.3  | 138.35| 181.33|       |      |      |     |
| 4.68      |     | 141.2 | 110.0 | 140.9 | 110.4 | 141.7 | 110.3 | 141.5 | 141.5 | 136.3 | 31.08| 22.81|     |
|           |     | 181.5 | 31.05 | 181.5 | 30.9  | 181.5 | 31.3  | 136.4 | 181.5 |       |      |      |     |
| 4.68      |     | 141.5 | 110.4 | 142.0 | 110.5 | 142.2 | 110.8 | 142.1 | 141.6 | 136.53| 31.43| 23.02|     |
| 4825      |     | 182.5 | 31.35 | 183.0 | 31.6  | 182.5 | 31.25 | 136.65| 182.67|       |      |      |     |

Comparison the magnetic field of the center of the resonance. Thus the quenching was measured several times at the nominal center of the resonance to ensure that the rf power setting was approximately right. Values of $\frac{d\phi}{dI_z}$ were obtained so that all data might be adjusted to the desired rf power level. The slope $\frac{d\phi}{dR}$ was measured at the working level so that the center of the resonance could be calculated if $\phi_a$ was not exactly equal to $\phi_a$. In cases of transitions αe and αe a measure of the small amount of β-component in the beam was obtained at several magnetic fields (see Sec. 73). One or more calibrations of permanent magnet were made by observing the sharp resonance $H(αβ)$. Temperatures in the vicinity of electro- and permanent-magnets were recorded at frequent intervals during the run. Various pressures, oven and bombarding currents and voltages, etc., were recorded, although they were not needed in analysis of the data. The small difference of the crystal frequency standard from WWV was determined several times during the year and interpolation was used for the day in question.

76. Corrections Applied in Calculation of a Run

(a) Soft component correction.—If appreciable contamination of the beam by β-state was expected, a measurement of

$$s = \frac{\phi_β}{\phi_e}$$  \hspace{1cm} (250)

was made at the two magnetic fields $H_e$ and $H_β$ of the working points, at the nominal central field $H_a$ and, for reference, also at 1450 gauss where a large value of $s$ is obtained. The method of making these measurements was indicated in Sec. 73. Values of $\phi$ obtained during the run must be corrected by a factor $1 + s$.

(b) Power variation correction.—The current crystal was measured each time the rf was applied. Since the variations were small, it sufficed to associate the average crystal current with the average percentage of quenching for each row of data. The value of $\frac{d\phi}{dI_z}$ was measured for the working point level and was calculated with sufficient accuracy for other levels by Eq. (102). The height of the nominal center was then brought to the desired level of quenching, e.g., 31 percent for $H(αβ)$, thereby determining a standard value of crystal current for the run. The remaining data were then reduced to this crystal current.

(c) Frequency variation corrections.—All quenchings were then adjusted to the same frequency, e.g., 2195 Mc/sec for $H(αβ)$, by applying a correction

$$\delta \phi = \pm \left( \frac{d\phi}{dR} \frac{R}{H} \frac{dI}{dH} \right) \delta f$$  \hspace{1cm} (251)

to the quenching, where $R/H$ is given by the magnetic field calibration, $dI/dH$ is the calculated slope of the Zeeman frequency versus magnetic field curves, and $\delta f = 5.00 - \Delta f$. The + and − signs are taken for points on the low and high field sides of the resonance, respectively.

77. Determination of Center

Denoting the average of both $\phi$'s for $R_a$ by $\bar{\phi}_a$ and likewise for $R_b$, one finds that the nominal center corresponds to a resistance $\bar{R}$,

$$\bar{R} = \frac{1}{2}(R_a + R_b) + \frac{1}{2} \left[ \bar{\phi}_a - \bar{\phi}_b \right] \left( \frac{d\phi}{dR} \right)$$  \hspace{1cm} (252)

where corrected values of $R_a$ and $R_b$ are used. From $\bar{R}$ one may calculate the apparent central field $\bar{H}$ by use of the field calibration figure $R/H$ for the day in question.

To illustrate the calculations outlined above, and to indicate the magnitude of the corrections, a sample calculation of central field is given in Table X for the data presented in Table IX. The necessary auxiliary information shown in the table has been obtained from measurements not indicated in Table IX. It should be recalled that a "run" consisted primarily of about
Table X. Calculation of sample data from run of 7/13/50 on transition \(H(a\beta)\). The table gives crystal current calculated to give desired central quenching, dependence of quenching on crystal current, magnetic field calibration \(R/H\), correction for departure of frequency from 2195 Mc/sec. Crystal current and frequency corrections are then calculated. In the last part of the table, the magnetic field of the resonance center is computed. For this, the slope in percent per ohm, the difference in quenching \(\Delta \phi\) on the two sides of the resonance and the field calibration are combined to give a resonance field of \(H_e = 704.27\) gauss.

<table>
<thead>
<tr>
<th>(R_{\text{nom}})</th>
<th>(R_{\text{total}})</th>
<th>(\Delta \phi/\Delta H = 0.004076)</th>
<th>(\phi_0)</th>
<th>(\phi_1)</th>
<th>(\phi_1)</th>
<th>(\phi_0)</th>
<th>(\Delta \phi)</th>
<th>(\phi_1)</th>
<th>(\phi_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4162</td>
<td>4274.57</td>
<td>20.47</td>
<td>180.67</td>
<td>1.99</td>
<td>0.17</td>
<td>20.30</td>
<td>0.32</td>
<td>0.08</td>
<td>20.38</td>
</tr>
<tr>
<td>4846</td>
<td>4958.63</td>
<td>22.92</td>
<td>182.09</td>
<td>3.41</td>
<td>0.32</td>
<td>22.60</td>
<td>0.32</td>
<td>-0.08</td>
<td>22.52</td>
</tr>
</tbody>
</table>
\(A_v = 4616.60\)

\[
\frac{\Delta \phi}{\Delta R} = 0.06762 \text{ at } R_{\text{nom}} = 4162 \text{ ohm}
\]
\[
\Delta \phi = 22.52 - 20.38 = 2.14 \text{ percent}
\]
\[
\Delta R = \left(\frac{2.14}{0.06762}\right) = 15.82 \text{ ohm}
\]
\(R_e = 4632.42 \text{ ohm} \quad H_e = \frac{4632.42}{6.577649} = 704.27 \text{ gauss}\)

fifteen such comparisons. Table XI shows the distribution of central fields obtained from the entire run devoted to transition \(H(a\beta)\), 2195 Mc/sec of 7/13/50. The average field center is \(H = 704.06\) gauss with an average deviation (av dev) = 0.06 gauss. This deviation gives a measure of the internal consistency of the data, but does not show external errors due to various causes discussed below.

78. Summary of Central Field Measurements

The results of all fifteen runs are given in Tables XII and XIII, together with a number of other quantities which are of significance in judging the accuracy of the data. The runs on transitions \(a\alpha\) and \(a\epsilon\) differ from those on \(a\beta\) because for the former a correction for \(\beta\)-component is necessary, while instead for \(a\beta\) overlap from \(a\epsilon\) must be considered. Columns headed \(s_0\) and \(s_\beta\) show the percentages of \(\beta\)-state in the beam for the working points, while under \(\phi_0\) and \(\phi_{300}\), respectively, are indicated the amounts of \(\beta\) quenching at the nominal center and 930 gauss. The latter field lies between the \(a\beta\) and \(a\epsilon\) peaks, and the value of \(\phi_{300}\) serves to determine the effect of overlap from \(a\epsilon\).

Q. ASYMMETRY AND SHIFT CORRECTIONS

79. Asymmetry and Stark Effect Corrections

Although the resonance curves are highly symmetrical there are a number of small effects leading to slight asymmetries which must be allowed for if the level shifts are to be determined to the desired accuracy. These have been listed in Sec. 45 and equations for their calculation given in Part III and Sec. 74. Since the completion of Part III, it was realized that the velocity distribution in the quenching function \(G(\psi)\) should have had a \(\psi^2\) instead of \(\psi^4\) dependence. This defect will be corrected in connection with later more accurate work. Fortunately, for our purposes, the effect of this error will be negligible except in the case of Stark effect. The correction to \(S\) should be reduced by a factor closely approximating one-half, and the calculations of this paper are modified accordingly. There are also somewhat smaller changes in effects of quenching asymmetry, but since these corrections are small, the changes can be neglected here.

The resulting corrections for the case of transition \(H(a\epsilon)\) at 2195 Mc/sec with \(\phi_\epsilon = 31\) percent are given in Table XIV. (There is no overlap correction here, and the \(\beta\)-component correction was made at the earlier stage indicated by Table X.) Each \(\Delta \phi\) is the calculated difference in quenching on opposite sides produced by the indicated asymmetry. The corresponding contribution to the expected frequency of the transition is

Table XI. Results of run on \(H(a\beta)\) of 7/13/50 based on calculations like those given in Table X for the first two lines of data.

<table>
<thead>
<tr>
<th>(R_e = 4274.57 \text{ ohm})</th>
<th>(R_0 = 4958.63 \text{ ohm})</th>
<th>(R_{300} = 4616.60 \text{ ohm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_a)</td>
<td>(R_b)</td>
<td>(R_c)</td>
</tr>
<tr>
<td>20.380</td>
<td>22.522</td>
<td>2.142</td>
</tr>
<tr>
<td>20.535</td>
<td>22.399</td>
<td>1.864</td>
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<td>22.402</td>
<td>22.398</td>
<td>1.929</td>
</tr>
<tr>
<td>20.469</td>
<td>20.376</td>
<td>2.022</td>
</tr>
<tr>
<td>20.416</td>
<td>22.333</td>
<td>1.957</td>
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<td>22.404</td>
<td>22.326</td>
<td>1.988</td>
</tr>
<tr>
<td>22.396</td>
<td>20.388</td>
<td>2.008</td>
</tr>
<tr>
<td>Average</td>
<td>20.434%</td>
<td>2.393%</td>
</tr>
<tr>
<td>Av dev</td>
<td>0.059%</td>
<td>0.034%</td>
</tr>
<tr>
<td>(R/H = 6.577649 \text{ ohm/gauss})</td>
<td>(H_e = 704.06 \text{ gauss})</td>
<td></td>
</tr>
</tbody>
</table>

(Av dev) = 0.06 gauss
TABLE XII. Summary of runs on $ae$ and $ae$. The table gives separation in gauss of working points, average quenching for these, magnetic field calibration $R/H$. The measured slopes $d\phi/dH$ are based on relatively little data, since they are needed only for adjusting $\phi_a$ and $\phi_e$ to the same level. The calculated quenching and slope values are given for comparison. The columns $s_a$ and $s_e$ give percentages of soft component at the working points, while the last two columns contain average magnetic fields for the various runs together with the average deviations of individual values from these averages.

<table>
<thead>
<tr>
<th>Date</th>
<th>$B_a-B_e$ gauss</th>
<th>$\phi_%$</th>
<th>$R/H$</th>
<th>$d\phi/dH$</th>
<th>$s_a$%</th>
<th>$s_e$%</th>
<th>$H_a$ gauss</th>
<th>$\Delta \phi_{dev}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition $H(ae)$ (2915 Mc/sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/27</td>
<td>114.47</td>
<td>20.87</td>
<td>6.57076</td>
<td>0.3218</td>
<td>...</td>
<td>...</td>
<td>1159.11</td>
<td>0.14</td>
</tr>
<tr>
<td>3/17</td>
<td>114.81</td>
<td>21.15</td>
<td>6.55112</td>
<td>0.3463</td>
<td>2.0</td>
<td>1.6</td>
<td>1159.41</td>
<td>0.19</td>
</tr>
<tr>
<td>3/25</td>
<td>114.78</td>
<td>20.99</td>
<td>6.53277</td>
<td>0.2932</td>
<td>0.2</td>
<td>0.1</td>
<td>1158.70</td>
<td>0.25</td>
</tr>
<tr>
<td>4/28</td>
<td>115.70</td>
<td>20.76</td>
<td>6.55310</td>
<td>0.3162</td>
<td>0.0</td>
<td>0.0</td>
<td>1158.94</td>
<td>0.20</td>
</tr>
<tr>
<td>Calculated</td>
<td>115.25</td>
<td>20.92</td>
<td></td>
<td>0.310</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition $D(ae)$ (2915 Mc/sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/14</td>
<td>84.20</td>
<td>20.01</td>
<td>6.56792</td>
<td>0.3411</td>
<td>2.7</td>
<td>0.6</td>
<td>1158.73</td>
<td>0.07</td>
</tr>
<tr>
<td>2/23</td>
<td>84.15</td>
<td>20.10</td>
<td>6.57110</td>
<td>0.3642</td>
<td>1.3</td>
<td>0.8</td>
<td>1158.71</td>
<td>0.09</td>
</tr>
<tr>
<td>5/2</td>
<td>84.25</td>
<td>19.89</td>
<td>6.55256</td>
<td>0.3398</td>
<td>0.6</td>
<td>0.4</td>
<td>1159.07</td>
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</tr>
<tr>
<td>Calculated</td>
<td>84.29</td>
<td>19.92</td>
<td></td>
<td>0.343</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition $D(ae)$ (7195 Mc/sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8/14</td>
<td>35.80</td>
<td>22.85</td>
<td>6.58096</td>
<td>0.9154</td>
<td>6.1</td>
<td>5.9</td>
<td>1188.94</td>
<td>0.10</td>
</tr>
<tr>
<td>10/19</td>
<td>35.78</td>
<td>22.12</td>
<td>6.58438</td>
<td>0.7552</td>
<td>0.6</td>
<td>0.6</td>
<td>1188.98</td>
<td>0.09</td>
</tr>
<tr>
<td>Calculated</td>
<td>35.82</td>
<td>21.66</td>
<td></td>
<td>0.742</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

given by

$$\delta F = \Delta F \left/ \left( \frac{|d\phi|}{|df|} \right) \right. \tag{253}$$

The total correction is $\Delta F = 1.34 \text{ Mc/sec}$ and the effect $\Delta \delta$ on the desired level shift $\delta$ is just the negative of this.

A summary of such corrections to the frequencies of all five transitions is given in Table XV.

80. Miscellaneous Corrections

For various reasons no temperature corrections were made. The most serious error involved is caused by reversible changes of magnetization of the permanent magnet due to temperature changes during a run. The method contemplated for these corrections was somewhat defective and must be improved in future work. However, the possible order of magnitude of error due to this cause is well below the 0.5 Mc/sec level and is allowed for in the limits of error given for the final results.

Corrections for departure of the frequency standard from 5 Mc/sec are necessary. If the true frequency is $(1+\epsilon)$ times the nominal frequency, then all microwave oscillator frequencies are increased by a factor $(1+\epsilon)$. Such a factor must also be applied to all magnetic fields as they are determined by Eq. (164) in terms of a measured frequency. Because of the near linearity of the Zeeman splitting with magnetic field, all results for $\delta$ and $\Delta E - \delta$ must be multiplied by the same factor $(1+\epsilon)$. The correction amounts to about 0.08 Mc/sec for $\delta$ and about 0.6 Mc/sec for $\Delta E - \delta$.

The error in field calibration due to the Stark effect produced by distant levels was shown in Sec. 57 to be negligible. Since the calibrations were made at 1613.5 to 1615 Mc/sec, while the crossing point for $\beta$ and $\epsilon$ is now known to be nearer 1610 Mc/sec, there is a slight error in field determination which can be estimated from Eq. (219) (see Fig. 53). This might amount to a correction to $\delta$ at most of order 0.1 Mc/sec, and will be neglected. Care will be taken in future work to calibrate much nearer the crossing point. Of course it was not possible to calculate this point exactly until a fairly accurate value for $\delta$ was available.

TABLE XIII. Summary of runs on transitions of. Similar to Table XII except that no soft component is present in beam. Instead, a measure of overlap from the $ae$ peak is given in columns $\phi_a$ and $\phi_e$.

<table>
<thead>
<tr>
<th>Date</th>
<th>$B_a-B_e$</th>
<th>$\phi_%$</th>
<th>$R/H$</th>
<th>$(d\phi/dH)_a$</th>
<th>$\phi_%$</th>
<th>$\phi_e$%</th>
<th>$\bar{H}_a$ gauss</th>
<th>$\Delta \phi_{dev}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition $H(ae)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7/10</td>
<td>103.85</td>
<td>21.30</td>
<td>6.56780</td>
<td>0.5238</td>
<td>28.3</td>
<td>5.53</td>
<td>704.26</td>
<td>0.12</td>
</tr>
<tr>
<td>7/12</td>
<td>104.04</td>
<td>21.40</td>
<td>6.57522</td>
<td>0.4672</td>
<td>28.3</td>
<td>5.78</td>
<td>704.02</td>
<td>0.11</td>
</tr>
<tr>
<td>7/13</td>
<td>104.00</td>
<td>21.42</td>
<td>6.57765</td>
<td>0.4448</td>
<td>28.3</td>
<td>5.82</td>
<td>704.07</td>
<td>0.06</td>
</tr>
<tr>
<td>10/27</td>
<td>101.78</td>
<td>21.90</td>
<td>6.58308</td>
<td>0.5160</td>
<td>28.3</td>
<td>5.89</td>
<td>704.53</td>
<td>0.17</td>
</tr>
<tr>
<td>Calculated</td>
<td>103.98</td>
<td>22.11</td>
<td></td>
<td>0.588</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Transition $D(ae)$</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7/14</td>
<td>46.89</td>
<td>23.51</td>
<td>6.57072</td>
<td>0.5909</td>
<td>33.26</td>
<td>3.84</td>
<td>704.32</td>
<td>0.08</td>
</tr>
<tr>
<td>8/1</td>
<td>46.83</td>
<td>23.86</td>
<td>6.58754</td>
<td>0.5721</td>
<td>33.19</td>
<td>3.96</td>
<td>704.37</td>
<td>0.07</td>
</tr>
<tr>
<td>Calculated</td>
<td>46.84</td>
<td>22.57</td>
<td></td>
<td>0.638</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table XIV. Some details of calculation of theoretical asymmetry and shift corrections for transition $H(\alpha e)$. The theory required has been given in Part III.

<table>
<thead>
<tr>
<th>Nominal frequency 2195 Mc/sec. Nominal magnetic field of resonance center 1159 gauss.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constants for undistorted resonance</td>
</tr>
<tr>
<td>$a_1 = 28.388$, $a_4 = 28.388$ Mc/sec</td>
</tr>
<tr>
<td>$b = 49.846$ Mc/sec, $A = 0.57918$</td>
</tr>
<tr>
<td>Working point frequency $\xi = 58.8$ Mc/sec</td>
</tr>
<tr>
<td>Calculated quenching $\phi = 20.917%$</td>
</tr>
<tr>
<td>Calculated slope $\frac{\partial \phi}{\partial \xi} = 0.3077%$/Mc/sec</td>
</tr>
<tr>
<td>Distortion due to variation of matrix element</td>
</tr>
<tr>
<td>$\phi(\xi) = 20.917%$, $\phi(-\xi) = 20.741%$</td>
</tr>
<tr>
<td>$\Delta \phi = \phi(\xi) - \phi(-\xi) = 0.356%$</td>
</tr>
<tr>
<td>Required correction to shift is $\Delta S = -0.577$ Mc/sec</td>
</tr>
<tr>
<td>Quenching asymmetry</td>
</tr>
<tr>
<td>$\phi(\xi) = 21.528%$, $\phi(-\xi) = 21.478%$</td>
</tr>
<tr>
<td>$\Delta \phi = 0.050%$</td>
</tr>
<tr>
<td>$\Delta S = -0.081$ Mc/sec</td>
</tr>
<tr>
<td>Incomplete Back-Goudsmith effect and Zeeman curvature</td>
</tr>
<tr>
<td>$a_1 = 29.197$, $a_4 = 27.579$ Mc/sec</td>
</tr>
<tr>
<td>$c_1 = 1.02079$, $c_4 = 1.020027$ Mc sec$^{-1}$ gauss$^{-1}$</td>
</tr>
<tr>
<td>$d_1 = 3.383 \times 10^{-3}$, $d_4 = 3.383 \times 10^{-3}$ Mc sec$^{-1}$ gauss$^{-2}$</td>
</tr>
<tr>
<td>$\phi(b = -57.5) = 21.283%$, $\phi(b = +57.5) = 20.675%$</td>
</tr>
<tr>
<td>$\Delta \phi = \phi(-57.5) - \phi(+57.5) = 0.563%$</td>
</tr>
<tr>
<td>$\Delta S = -0.915$ Mc/sec</td>
</tr>
<tr>
<td>Stark effect</td>
</tr>
<tr>
<td>$\Delta S = 0.230$ Mc/sec</td>
</tr>
<tr>
<td>Total asymmetry and shift correction to level separation</td>
</tr>
<tr>
<td>$\Delta S = -1.34$ Mc/sec</td>
</tr>
</tbody>
</table>

A calculation of the effect of contamination of the deuterium by hydrogen and a mass spectroscopic analysis$^{98}$ for hydrogen showed that only a negligible error could be due to this cause.

R. RESULTS

81. Quantities Which Can Be Determined

From each run on $ae$ and $af$ we have now determined a magnetic field $H_0$. If there were no level shift $S$, one could calculate a theoretical frequency $F(H_0)$ from Eqs. (164)–(172) with $y_0 = 0$. If the actual frequency is $f$, the level shift is given by $S = f - F(H_0)$ to which should be added the correction $\Delta S$ from Table XV as well as the miscellaneous corrections of Sec. 80. For the low frequency transitions, the result obtained for $S$ is highly independent of the assumed value of $\Delta E$. For transition $D(\alpha e)$, however, only the combination $\Delta E - S$ is really determined. By using the adopted value of $S$ it is possible to calculate $\Delta E$ and, if Eq. (135) is assumed, to determine a value for the fine structure constant $\alpha$.

82. Weights Assigned to Data

As will be seen from Table XVI, the average deviation (av dev) for a single run is usually somewhat smaller than the differences in $S$ obtained from different runs on a given transition. The question arises whether one should give each run an equal weight or should assign weights proportional to the number of lines of data of the run and inversely proportional to (av dev)$^3$. The correct procedure depends on the kinds of errors which lead to the discrepancies, and these are not known. Fortunately, the results obtained by the two methods differ at most by 0.1 Mc/sec except in case of transition $H(\alpha f)$ where the run of 10/27/50, having a very low weight, accounts for the larger difference. Rather arbitrarily, the first method of assigning weights was adopted.

83. Results

The results are as follows: For hydrogen, the same value for the $2\Sigma_1^1 - 2\Sigma_1^1$ shift $S_0 = 1058.27$ Mc/sec is obtained from transitions $ae$ and $af$. For deuterium, the values of $S_0$ are 1059.81 and 1059.61, respectively, with average $S_0 = 1059.71$ Mc/sec. The limit of error is conservatively estimated as 1.0 Mc/sec for each isotope. The deuterium-hydrogen difference is $S_0 - S_0 = 1.44$ Mc/sec with limit of error set also at 1.0 Mc/sec, since it is felt that there should be some cancellation of errors.

Assuming the above value of $S_0$, the high frequency transition $D(\alpha e)$ can be used to calculate $\Delta E = 10,972.11$ Mc/sec with limit of error of 1.0 Mc/sec.

84. Discussion of Results

The agreement between values of $S$ obtained from transitions $ae$ and $af$ must be regarded as somewhat fortuitous. Of course, the results would be vitiated if some shift or resonance asymmetry had been omitted from the theory. The likelihood of this may perhaps be judged by comparing the observed and calculated quenching $\phi$ and slope $\partial \phi/\partial S$ values shown in Tables XII and XIII. The agreement is fairly good, but much better for $H(\alpha e)$ and $D(\alpha e)$ than for the other transitions. It is not known whether the discrepancies are of experimental$^{99}$ or theoretical origin. More accurate

Table XV. Summary of asymmetry and shift corrections to $S$ for transitions $ae$ and $af$, and to $\Delta E - S$ for transition $D(\alpha e)$.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$H(\alpha e)$</th>
<th>$D(\alpha e)$</th>
<th>$H(\alpha f)$</th>
<th>$D(\alpha f)$</th>
<th>$D(\alpha e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2195</td>
<td>2195</td>
<td>2395</td>
<td>2395</td>
<td>7193</td>
</tr>
<tr>
<td>Overlap correction</td>
<td>0</td>
<td>0</td>
<td>+0.984</td>
<td>+0.243</td>
<td>0</td>
</tr>
<tr>
<td>Matrix element variation</td>
<td>-0.577</td>
<td>-0.381</td>
<td>-0.279</td>
<td>-0.124</td>
<td>-0.156</td>
</tr>
<tr>
<td>Quenching asymmetry</td>
<td>-0.081</td>
<td>+0.031</td>
<td>-0.141</td>
<td>+0.015</td>
<td>-0.033</td>
</tr>
<tr>
<td>Incomplete Back-Goudsmith effect and Zeeman curvature</td>
<td>-0.915</td>
<td>-0.113</td>
<td>-2.474</td>
<td>-0.150</td>
<td>+0.004</td>
</tr>
<tr>
<td>Stark effect</td>
<td>+0.230</td>
<td>+0.115</td>
<td>-0.005</td>
<td>-0.032</td>
<td>+0.012</td>
</tr>
<tr>
<td>Forbidden component</td>
<td>0</td>
<td>0</td>
<td>+0.192</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total correction (Mc/sec)</td>
<td>-1.34</td>
<td>-0.35</td>
<td>-1.78</td>
<td>-0.05</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

$^{98}$ Since transitions $af$ and $ae$ occur in the fringing region of the rf field, it is possible that magnetic field inhomogeneity (Sec. 38) might contribute to the discrepancies. The indicated limits of error allow for this effect.

$^{99}$ This analysis was kindly made for us by Mr. R. Crom of Professor T. I. Taylor's laboratory.
measurements now in progress at Columbia may answer the question. Tests of this kind provide an indirect qualitative confirmation of the theory of hyperfine splitting for the 2p states.

Taking the above uncertainty and the sizes of the known corrections (as shown in Table XV) into account, it would appear that the values of \( \delta \) obtained from transitions (ae) would be more accurate than \( \delta \) values obtained from transitions af. Since the corrections are smaller for deuterium than for hydrogen, the shift for the former isotope should be the more accurate of the two.

The difference between the present values for \( \delta \) and the previously quoted 1062±5 Mc/sec\(^{100}\) can mostly be attributed to use of an improved theory of the hydrogen atom including effects due to reduced mass and the anomalous magnetic moment of the electron.

When the value of \( AE = 10,972.11 \) Mc/sec obtained above for deuterium is compared to the input value 10,970.45 Mc/sec, the following is seen: (a) There is very good evidence for the bulk of the 25.07 Mc/sec contribution to \( \Delta E \) arising from the anomalous magnetic moment of the electron, and (b) there appears to be a 1.66 Mc/sec discrepancy between observed and input values of \( \Delta E \). If the very well-founded theoretical formula (135) for \( \Delta E \) is assumed, a fractional increase in the value of the fine structure constant \( \alpha \) of 76 parts per million (ppm) would be required. The input value for \( 1/\alpha \) of 137.043 would accordingly emerge as 137.033 with a limit of error of ±0.006. Future work should give considerably better precision, but it is believed that the above represents a significant indication of the need for a thoroughgoing revision of physical constants related to \( \alpha \). Recent discussions by Bearden and Watts\(^{106}\) and by DuMond and Cohen\(^{107}\) determine \( \alpha \) primarily from the extremely accurate measurements of hyperfine splitting of the ground state of hydrogen by Proddell and Kusch.\(^{108}\) For this purpose they must use a theoretical formula connecting the hyperfine splitting with \( \alpha \). Radiative corrections to this formula have recently been given by Kroll and Pollock\(^{104}\) and others\(^{105}\) which in themselves would lead to \( 1/\alpha = 137.036 \), a change by more than seven times the probable error assigned by DuMond and Cohen, and ten times that given by Bearden and Watts. However, additional corrections due to spatial distribution of nuclear magnetism and reduced mass effects are still necessary. Until these can be made with confidence, the value of the fine structure constant will be more reliably determined from fine structure than from hyperfine structure. Ultimately, one may hope that \( \alpha \)-values determined by the two very different and independent methods will be in agreement.

The value of \( S = 1058.27 \) Mc/sec for hydrogen differs considerably from the value 1051.41 Mc/sec computed\(^{67}\) on the basis of second-order quantum electrodynamics. A number of corrections to this have now been calculated. Among these are \(-0.94 \) Mc/sec\(^{109}\) on account of the fourth-order anomalous magnetic moment of the electron, and a calculation by Baranger\(^{109}\) of contributions to \( \delta \) of order \( Z\alpha^4 \) amounting to 6.89 Mc/sec. Kroll\(^{108}\) has found corresponding vacuum polarization terms to be 0.25 Mc/sec. Bersohn, Weneser, and Kroll\(^{102}\) have estimated the higher order corrections of order \( Z\alpha^4 \) to be in the range 0.25±0.10 Mc/sec.

In order to calculate the resulting level shift for infinite nuclear mass, we must remove the reduced mass

\(^{100}\) Part II, Sec. 46.
\(^{105}\) N. M. Kroll and F. Pollock, Phys. Rev. 84, 594 (1951).

### Table XVI. Summary of corrections applied to data to obtain level shifts. All frequencies are in Mc/sec.

<table>
<thead>
<tr>
<th>Run</th>
<th>( H_\alpha ) (gusses)</th>
<th>( S ) (apparent)</th>
<th>Frequency correction</th>
<th>Asymmetry and shift correction</th>
<th>( S ) (final)</th>
<th>Av dev for run</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/27</td>
<td>1159.11</td>
<td>1059.46</td>
<td>+0.09</td>
<td>-1.34</td>
<td>1058.21</td>
<td>0.14</td>
</tr>
<tr>
<td>3/17</td>
<td>1159.41</td>
<td>1059.15</td>
<td>0.08</td>
<td>-1.34</td>
<td>1057.89</td>
<td>0.19</td>
</tr>
<tr>
<td>3/23</td>
<td>1158.70</td>
<td>1059.88</td>
<td>0.08</td>
<td>-1.34</td>
<td>1058.62</td>
<td>0.25</td>
</tr>
<tr>
<td>4/28</td>
<td>1158.94</td>
<td>1059.62</td>
<td>0.08</td>
<td>-1.34</td>
<td>1058.36</td>
<td>0.20</td>
</tr>
<tr>
<td>Av</td>
<td></td>
<td></td>
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<td></td>
<td>1058.27</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>( H_\alpha )</th>
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<th>7/12</th>
<th>7/13</th>
<th>10/27</th>
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<tbody>
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<td></td>
<td>704.26</td>
<td>1059.91</td>
<td>1060.38</td>
<td>1060.28</td>
<td>1059.39</td>
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<tr>
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<td>-1.78</td>
<td>-1.78</td>
<td>-1.78</td>
<td>-1.78</td>
</tr>
<tr>
<td></td>
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<td></td>
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<td></td>
<td>1058.20</td>
</tr>
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<td>1058.67</td>
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<td></td>
<td></td>
<td></td>
<td>1058.57</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>1057.66</td>
</tr>
<tr>
<td>Av</td>
<td></td>
<td>1058.27</td>
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<td></td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
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<th>( D_\alpha )</th>
<th>2/14</th>
<th>2/23</th>
<th>5/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1158.73</td>
<td>1060.18</td>
<td>1060.20</td>
<td>1059.07</td>
</tr>
<tr>
<td></td>
<td>+0.09</td>
<td>-0.35</td>
<td>-0.35</td>
<td>-0.35</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
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<td>1059.92</td>
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</tr>
<tr>
<td>Av</td>
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<td>1059.81</td>
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</table>

<table>
<thead>
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<th>7/14</th>
<th>8/1</th>
<th>10/19</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>704.32</td>
<td>1059.64</td>
<td>1059.54</td>
<td>1188.98</td>
</tr>
<tr>
<td></td>
<td>+0.07</td>
<td>-0.05</td>
<td>-0.05</td>
<td>9911.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.11</td>
</tr>
<tr>
<td></td>
<td>((AE - \delta))</td>
<td>((AE - \delta))</td>
<td>((AE - \delta))</td>
<td>9912.38</td>
</tr>
<tr>
<td>(apparent)</td>
<td></td>
<td>(final)</td>
<td>(final)</td>
<td>9912.41</td>
</tr>
<tr>
<td>Av</td>
<td></td>
<td>9912.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

correction \(-s/M = -0.575 \text{ Mc/sec}\) assumed by Bethe, Brown, and Stehn\(^{107}\) for hydrogen. A further correction of \(3 \times 76\) ppm or \(+0.24\) Mc/sec is needed because of the indicated change in value of \(1/\alpha\) from 137.043 to 137.033. The result is

\[
\begin{align*}
\bar{s} = & 1051.41 - 0.94 + 6.89 + 0.25 + 0.25 + 0.575 + 0.24 \\
= & 1058.67 \text{ Mc/sec}.
\end{align*}
\]

This figure cannot be compared with the experimental results for hydrogen and deuterium until it has been corrected for reduced mass effects.

According to indications of present theory, the shifts \(s_H\) and \(s_D\) are related to \(s\) for infinite nuclear mass by equations of the form

\[
\begin{align*}
\bar{s}_H &= s(1 - aM^{-1}) + 0.08, \\
\bar{s}_D &= s(1 - \frac{3}{4}aM^{-1}) + 0.74 + 0.04. 
\end{align*}
\tag{254}
\]

The factors \((1 - aM^{-1})\) and \((1 - \frac{3}{4}aM^{-1})\) express the form of reduced mass dependence. Bethe, Brown, and Stehn\(^{107}\) took the coefficient \(a = 1\). It is known that this proportionality of \(s\) to reduced mass is incorrect. According to an argument of Welton in which \(s\) depends on the probability of finding the electron at the nucleus, one might expect \(a = 3\). However, there are also radiative contributions from the proton. These are being considered by Gourary\(^{111}\) and by Salpeter.\(^{112}\) Since their values for \(a\) are not yet final, we shall determine a semi-empirical value from the observed \(s_D - s_H\) difference. The additive constant \(0.74\) Mc/sec in Eq. (254) represents the effect of finite deuteron size\(^{105,112}\) (the value 0.45 Mc/sec of Part III, Appendix VI was calculated with a zero-range deuteron wave function.) The remaining terms 0.08 and 0.04 Mc/sec express the reduced mass dependence of the denominator of the argument of Bethe's logarithm. It is assumed that no reduced mass enters the numerator \(mc^2\). No attempt is made to include estimates of possible mesonic effects (Appendix VI b). When the observed difference \(s_D - s_H = 1.44\) Mc/sec is inserted into Eqs. (254), one obtains \(a = 2.58\). With this semi-empirical determination of the reduced mass effect, one finds calculated values \(s_H = 1057.27\) and \(s_D = 1058.71\) Mc/sec. The observed values for \(s_H\) and \(s_D\) are 1.00 Mc/sec larger than these.

It would be premature to claim that this difference represents a real discrepancy between observation and theory. It is quite possible that further calculations on the basis of present quantum electrodynamics will improve the agreement. In view of the surprisingly large magnitude of the Baranger\(^{110}\) correction, it may be that contributions of order \(Z^3a^2\) and higher must yet be calculated in order to make possible a significant comparison between theory and experiment. An experimental test of this conjecture will be possible when more accurate measurements are available for both \(Z = 1\) and \(Z = 2\). Any residual deviation might be ascribed to a departure from the Coulomb law for electron and proton. An effect of this sort would arise from the presence of a meson cloud about the nucleus, but according to such estimates as have been made from the theory of the electron neutron interaction (Appendix VI b) would not suffice. On the other hand, the \(f\) field considered by Pais\(^{119}\) would give a contribution for a proton and not for a neutron.

The authors have benefited greatly from many valuable discussions of the theoretical aspects of this work with Professor N. M. Kroll, and of experimental questions with E. S. Dayhoff and S. Triebwasser, who are continuing the measurements.

\(^{111}\) B. S. Gourary, private communication.

\(^{112}\) E. E. Salpeter, Bull. Am. Phys. Soc. 27, No. 1, 20 (1952), and private communication.