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Quantum gates in mesoscopic atomic ensembles based on adiabatic passage and Rydberg blockade

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We present schemes for geometric phase compensation in an adiabatic passage which can be used for the implementation of quantum logic gates with atomic ensembles consisting of an arbitrary number of strongly interacting atoms. Protocols using double sequences of stimulated Raman adiabatic passage (STIRAP) or adiabatic rapid passage (ARP) pulses are analyzed. Switching the sign of the detuning between two STIRAP sequences, or inverting the phase between two ARP pulses, provides state transfer with well-defined amplitude and phase independent of atom number in the Rydberg blockade regime. Using these pulse sequences we present protocols for universal single-qubit and two-qubit operations in atomic ensembles containing an unknown number of atoms.

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Quantum information can be stored in collective states of ensembles of strongly interacting atoms [1]. This idea can be extended to encoding an entire register of qubits in ensembles of atoms with multiple ground states [2] which opens up the possibility of large quantum registers in a single atomic ensemble [3], or of coupling arrays of small ensembles in a scalable atom-chip-based architecture [4]. Quantum information based on ensembles can be realized more generally in any ensemble of strongly coupled spins [5]. Our proposal for implementing high-fidelity quantum gates in ensembles is thus of interest for several different implementations of quantum computing.

The enhanced coupling to the radiation field by a factor of $\sqrt{N}$, with N the number of atoms or spins, is useful for coupling matter qubits to single photons [6]. Combining photon coupling with local quantum gates in ensembles enables architectures with improved fidelity for quantum networking [7]. The use of ensemble qubits is also attractive for the deterministic loading of registers of single-atom qubits [8–10] and for realizing gates that act on multiple particles. All of these capabilities rely on high-fidelity quantum gate operations between collectively encoded qubits. However, due to the dependence of the Rabi frequency of oscillations between different collective states on the number of atoms as $\sqrt{N}$, it is difficult to perform gates with well-defined rotation angles in the situation where N is unknown [11,12]. Although there is recent progress in the nondestructive measurement of N with high accuracy [13], it remains an outstanding challenge to implement high-fidelity quantum logic gates without precise knowledge of N, particularly in the case of collectively encoded registers [2] where the effective value of N depends on the unknown quantum state encountered during a computation.

Adiabatic passage techniques [stimulated Raman adiabatic passage (STIRAP) and adiabatic rapid passage (ARP)] have been widely used for deterministic population transfer in atomic and molecular systems [14,15]. These techniques have been studied for quantum state control [16], qubit rotations [17], creation of entangled states [18], and for deterministic excitation of Rydberg atoms [19,20]. Although the STIRAP or ARP methods provide pulse areas with strongly suppressed sensitivity to the Rabi frequency Ω_N, and therefore suppressed sensitivity to N, the phase of the final state is in general still strongly dependent on N. A randomly loaded dipole trap follows a Poissonian distribution in the atom number, with relative fluctuations $1/\sqrt{N}$. Indeed, gate errors at the level of 10^{-3} can be achieved, but would require $\bar{N} \sim 4000$, and achieving full blockade for such a large ensemble remains an outstanding challenge.

In this Rapid Communication we propose double adiabatic sequences using either STIRAP or ARP excitation which remove the phase sensitivity, and can be used to implement gates on collectively encoded qubits without precise knowledge of N even for moderately sized ensembles.

Method for phase compensation. Our approach is shown in Fig. 1. The quantum register consists of individually addressed atomic ensembles in arrays of optical dipole traps or optical lattices [Fig. 1(a)]. The energy-level scheme for STIRAP and ARP is shown in Fig. 1(b). A sequence of two STIRAP pulses is produced with fields having Rabi frequencies Ω_1,Ω_2, and detuning δ from the intermediate state. In the regime of a strong Rydberg blockade, the first STIRAP (ARP) pulse deterministically prepares the ensemble in a collective state with a single Rydberg excitation, as we demonstrated in Ref. [20]. The second reverse STIRAP pulse, as shown in Fig. 1(c), returns the Rydberg atom back to the ground state. A similar scheme can be implemented using linearly chirped ARP pulses, as shown in Fig. 1(d).

We have studied the population and phase dynamics of the collective states of the atomic ensemble interacting with laser

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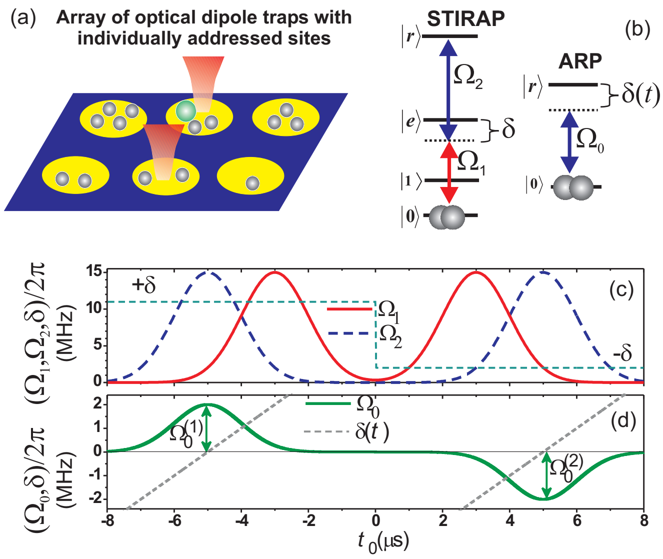

FIG. 1. (Color online) (a) Scheme of the quantum register based on individually addressed atomic ensembles in an array of optical dipole traps. Laser pulses are used to excite atoms into the Rydberg state. Only one atom in each site can be excited due to the Rydberg blockade. Simultaneous excitation of Rydberg atoms in the neighboring sites is also blocked. (b) Energy levels for two-photon STIRAP and single-photon ARP excitation. (c) Time sequence of STIRAP laser pulses. (d) Time sequence for ARP laser excitation.

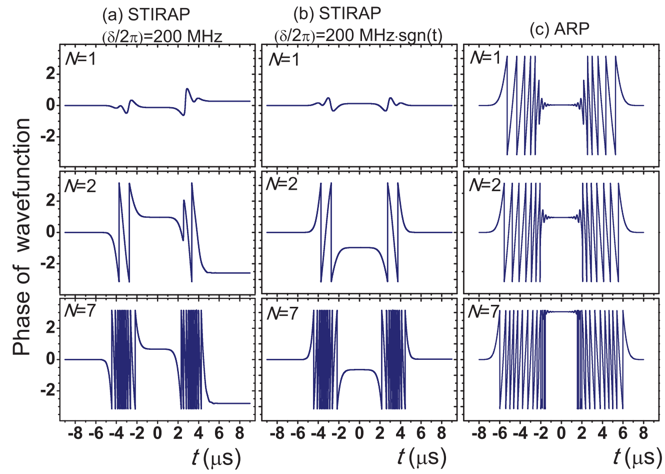

FIG. 2. (Color online) Calculated time dependence of the phase of the collective ground-state amplitude for $N = 1,2,7$ atoms (top to bottom). Double STIRAP sequence ($\Omega_1/2\pi = 30$ MHz, $\Omega_2/2\pi = 40$ MHz) (a) with $\delta/2\pi = 200$ MHz, (b) with $\delta/2\pi = 200$ MHz $\times$ sgn(t), and (c) for a double ARP pulse sequence with phase inversion. The single STIRAP sequence used $\Omega_j(t) = \Omega_j e^{-(t+t_j)^2/2\tau^2}$ for $j = 1,2$ with $\Omega_1/2\pi = 30$ MHz, $\Omega_2/2\pi = 40$ MHz, $t_1 = 3.5$ μs, $t_2 = 5.5$ μs, $\tau = 1$ μs, and $\delta/2\pi = 200$ MHz. The single ARP pulse used $\Omega_0(t) = \Omega_0 e^{-t^2/2\tau^2}$ with $\Omega_0/2\pi = 2$ MHz, $\tau = 1$ μs, and linear chirp $\alpha/2\pi = (1/2\pi)[d\delta(t)/dt] = 1$ MHz/μs [20].

radiation. Calculations were performed using the Schrödinger equation, neglecting spontaneous emission, and assuming a perfect blockade so only states with at most a single Rydberg excitation were included. The details of our calculations are discussed in the Supplemental Material [21]. At the end of a double STIRAP sequence the population is returned back to the collective ground state $|000\ldots\rangle$ of the atomic ensemble, but a geometric phase is accumulated. This phase shift of the ground state is dependent on the Rabi frequency and leads to gate errors. We have found that the phase of the atomic wave function can be compensated by switching the sign of the detuning between two STIRAP pulses, or by switching the phase between two ARP pulses, as shown in Fig. 2. For a double STIRAP sequence with the same detuning throughout, the accumulated phase depends on N [Fig. 2(a)], while the phase change is zero, independent of N, when we switch the sign of detuning δ between the two STIRAP sequences [Fig. 2(b)]. A similar phase cancellation occurs for π phase-shifted ARP pulses [Fig. 2(c)], which can be implemented using an acousto-optic modulator [22].

The probability of loading N noninteracting atoms in a small optical or magnetic trap is described, in general, by Poissonian statistics. For $\bar{N} = 5$ the probability to load zero atoms is 0.0067, which is small enough to create a large quantum register with a small number of defects [23]. Figure 3(a) shows a comparison of the fidelity of single-atom excitation for a single-photon π rotation with the area optimized for $N = 5$ atoms compared to STIRAP or ARP pulses. We see that the adiabatic pulses reduce the population error by up to several orders of magnitude for a wide range of N. Finite lifetimes of the intermediate excited state and Rydberg states can lead, however, to a breakdown of the deterministic excitation. Figure 3(b) shows the population errors for a single STIRAP sequence in the ensemble of $N = 1$–4 atoms with a linewidth of the intermediate state $\gamma_e/(2\pi) = 5$ MHz and of the Rydberg state $\gamma_r/(2\pi) = 0.8$ kHz calculated using density-matrix equations for an ensemble of interacting atoms [24] for two different detunings from the intermediate state

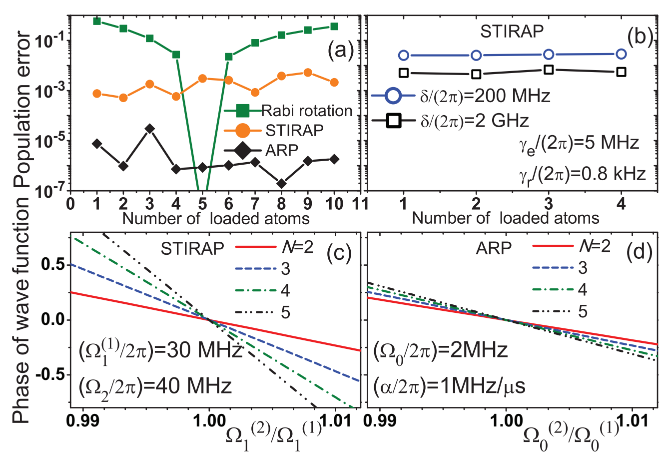

FIG. 3. (Color online) (a) Comparison of the fidelity of single-atom excitation by a π laser pulse having the area optimized for $N = 5$ atoms ($t = \pi/\sqrt{5}\Omega$), with a STIRAP sequence, and with an ARP pulse. All parameters are as in Fig. 2. Spontaneous emission is not taken into account. (b) The population error for a single STIRAP sequence calculated taking into account a linewidth $\gamma/(2\pi) = 5$ MHz of the intermediate state for detuning $\delta = 200$ MHz ($\Omega_1/2\pi = 30$ MHz, $\Omega_2/2\pi = 40$ MHz, $\tau = 1$ μs) and $\delta = 2$ GHz ($\Omega_1/2\pi = 250$ MHz, $\Omega_2/2\pi = 250$ MHz, $\tau = 0.2$ μs). (c),(d) Dependence of the phase error on Rabi frequency changes between pulses for STIRAP or ARP pulses calculated using the Schrödinger equation.

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$\delta = 200$ MHz ($\Omega_1/2\pi = 30$ MHz, $\Omega_2/2\pi = 40$ MHz, $\tau = 1\ \mu$s) and $\delta = 2$ GHz ($\Omega_1/2\pi = 250$ MHz, $\Omega_2/2\pi = 250$ MHz, $\tau = 0.2\ \mu$s). We see that the effects of the finite lifetime of the intermediate state are negligible if the detuning from the intermediate state is chosen so that $\Delta \gg \Omega$.

Although the proposed double-pulse sequences are almost insensitive to moderate variations of the absolute Rabi frequency, the main sources of errors are fluctuations of the Rabi frequencies between the first and second pulses. For perfectly identical pulses the population transfer error in ensembles of $N = 5$ atoms can be kept below 10^{-3} for STIRAP and below 10^{-4} for an ARP pulse for a wide range of Rabi frequencies. The dependence of the phase errors on parameters of the laser pulses are shown in Figs. 3(c) and 3(d). The dependence of the phase error on the ratio of Rabi frequencies $\Omega_1^{(2)}/\Omega_1^{(1)}$ between pulses [see Fig. 1(b)] is shown in Fig. 3(c) for $N = 1$–5 atoms. The single-photon ARP excitation in Fig. 3(d) demonstrates reduced sensitivity to fluctuations of the Rabi frequency and has higher efficiency at lower Rabi frequencies. Although this could be an important advantage over STIRAP, implementation of single-photon Rydberg excitation is difficult due to the need for ultraviolet laser radiation and a larger sensitivity to Doppler broadening [25]. For either approach the double-pulse amplitudes must be well matched for low phase errors. Using the fiber delay line, amplitude matching at the level of 10^{-6} is feasible over a time scale of a few microseconds [26].

Gates. We have developed protocols to implement quantum logic gates using phase-compensated double STIRAP or ARP. Consider atoms with levels $|0\rangle,|1\rangle,|e\rangle|r\rangle$ as shown in Fig. 1. A qubit can be encoded in an N atom ensemble with the logical states $|\bar{0}\rangle = |000\ldots000\rangle$, $|\bar{1}\rangle' = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}|000\ldots1_j\ldots000\rangle$. Levels $|0\rangle,|1\rangle$ are atomic hyperfine ground states. Coupling between these states is mediated by the singly excited Rydberg state $|\bar{r}\rangle' = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}|000\ldots r_j\ldots000\rangle$. A Rydberg blockade only allows single excitation of $|r\rangle$ so the states $|\bar{0}\rangle$ and $|\bar{r}\rangle'$ experience a collectively enhanced coupling rate $\Omega_N = \sqrt{N}\Omega$. States $|\bar{r}\rangle'$ and $|\bar{1}\rangle'$ are coupled at the single-atom rate Ω. State $|\bar{1}\rangle'$ is produced by the sequential application of π pulses $|\bar{0}\rangle \to |\bar{r}\rangle'$ and $|\bar{r}\rangle' \to |\bar{1}\rangle'$.

Pulse areas independent of N on the $|0\rangle \leftrightarrow |r\rangle$ transition can be implemented with STIRAP or ARP as described above. We will define the logical basis states and the auxiliary Rydberg state as $|\bar{0}\rangle = |000\ldots000\rangle$, $|\bar{1}\rangle = e^{\imath\chi_N}|\bar{1}\rangle'$, and $|\bar{r}\rangle = e^{\imath\chi_N}|\bar{r}\rangle'$. Here χ_N is the phase produced by a single N atom STIRAP pulse with positive detuning. We assume that we do not know the value of N, which may vary from qubit to qubit, and therefore χ_N is also unknown, but has a definite value for fixed N.

We find that arbitrary single-qubit rotations in the basis $|\bar{0}\rangle,|\bar{1}\rangle$ can be performed with high fidelity, without precise knowledge of N, by accessing several Rydberg levels $|r_0\rangle, |r_1\rangle$ as shown in Fig. 4(a). The equations which describe the gate sequence are discussed in the Supplemental Material [21]. The final state $|\psi\rangle = a'|\bar{0}\rangle + b'|\bar{1}\rangle$ is arbitrary and is selected by the rotation $R(\theta,\phi)$, in step 3: $\binom{a'}{-b'} = \mathbf{R}(\theta,\phi)\binom{a}{b}$. Depending on the choice of implementation, to be discussed below, this may be given by a one- or two-photon microwave pulse, with Rabi frequency Ω_3. Provided states $|r_0\rangle,|r_1\rangle$ are strongly interacting, and limit the number of excitations in the ensemble to one, the indicated sequence is obtained. In the regime of Ω_3 large compared to the Rydberg excitation rates, the time spent populating a Rydberg level corresponds to 4π of the Rydberg pulse area. This is the same as for a single-atom C_Z gate, and we therefore expect the limit on gate infidelity to be $\sim$0.002 [27] for small ensembles. It was shown in Ref. [3] that in a three-dimensional (3D) lattice the number of atoms N which can be entangled at fixed error scales linearly. Although the details of the error scaling are different for ensemble qubits, for moderately sized ensembles we anticipate approximately linear scaling, with a numerical prefactor that requires a detailed analysis to be given elsewhere.

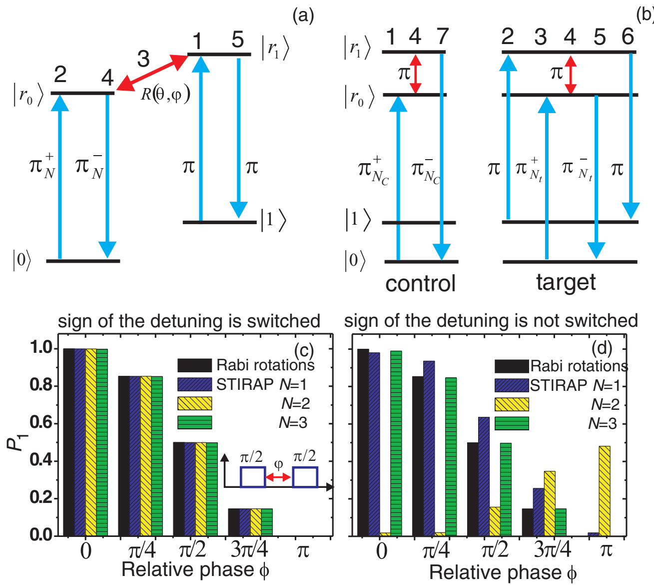

FIG. 4. (Color online) (a) Single-qubit gate for a mesoscopic qubit with N atoms. Pulses 1–5 act between the qubit states $|0\rangle,|1\rangle$ and the Rydberg states $|r_0\rangle,|r_1\rangle$. Pulses 1,2,4,5 are optical transitions and pulse 3 is a microwave frequency transition between Rydberg states. (b) CNOT gate between mesoscopic qubits with N_c atoms in the control qubit and N_t atoms in the target qubit. (c),(d) The dependence of the population of the qubit state $|1\rangle$ after two sequential $\pi/2$ rotations on the phase difference ϕ between the pulses (c) with and (d) without switching the sign of the detuning between the STIRAP sequences.

The five-pulse sequence we describe here is more complicated than the three pulses needed for an arbitrary single-qubit gate in the approach of Ref. [1]. The reason for this added complexity is that the special phase preserving the property of the double STIRAP or ARP sequences requires that all population is initially in one of the states connected by the pulses. The sequence of pulses in Fig. 4(a) ensures that this condition is always satisfied.

To verify that our scheme preserves coherence, we have numerically modeled the sequence of two single-qubit rotations for an angle of $\pi/2$ with relative phases ϕ in the range 0–π. The probability of finding the ensemble in the qubit state $|1\rangle$ was calculated for our STIRAP-based protocol for $N = 1$–3 atoms and compared with the outcome of a similar single-atom gate sequence applied using conventional Rabi rotations [shown as black in Fig. 4(c)]. We have found that the probability for the ensemble to be in state $|1\rangle$ is independent of the number of atoms and correctly depends on the relative phase between the microwave pulses, as shown in Fig. 4(c). In contrast, if we do not switch the detuning from the intermediate state after the first STIRAP pulse, the probability of finding the ensemble in the state $|1\rangle$ becomes N dependent and is inconsistent with the expected values, as shown in Fig. 4(d).

A controlled-NOT (CNOT) gate can be implemented by the sequence $H(t)$-C_Z-$H(t)$ [28], where the Hadamard gates are performed as in Fig. 4(a). The C_Z operation is implemented in analogy to schemes for single-atom qubits [29] mediated by Rydberg interactions, using the protocol $\pi_{|\bar{1}\rangle-|\bar{r}\rangle}$(c) $2\pi_{|\bar{1}\rangle-|\bar{r}\rangle}$(t) $\pi_{|\bar{1}\rangle-|\bar{r}\rangle}$(c), where c (t) stand for control (target) qubits. The CNOT gate therefore requires a total pulse area of 12π Rydberg pulses. We can reduce this to 7π of Rydberg pulses as shown in Fig. 4(b) which implements an approach analogous to the amplitude-swap gate demonstrated for single-atom qubits in Ref. [30]. All pulses except number 4 in the sequence are optical and are localized to either the control or target qubit. Pulse 4 is a microwave field and drives a π rotation on both qubits. As for the single-qubit gate the requirement for high-fidelity operation is that the interactions $|r_0\rangle \leftrightarrow |r_0\rangle$, $|r_1\rangle \leftrightarrow |r_1\rangle$, $|r_0\rangle \leftrightarrow |r_1\rangle$ all lead to a full blockade of the ensembles, and we refer to the Supplemental Material [21] for the choice of n that fulfills this condition. Since the frequency of pulse 4, which is determined by the energy separation of states $|r_0\rangle, |r_1\rangle$, can be chosen to be very different from the qubit frequency given by the energy separation of states $|0\rangle, |1\rangle$, the application of microwave pulses will not lead to crosstalk in an array of ensemble qubits.

In summary we have demonstrated that double STIRAP and ARP sequences with phase compensation enable high-fidelity quantum gates in collectively encoded ensembles. We have shown that phase compensation using this method works effectively regardless of the number of atoms N even in small atomic ensembles randomly loaded, which display a large fractional variation in N. We have presented full protocols for one-qubit and two-qubit logic gates which perform at high fidelity both in the regime of small and large ensembles. We anticipate that these ideas will contribute to the realization of quantum logic using collectively encoded qubits and registers.

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[21] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.88.010303 for the schemes of the quantum gates, details of the numeric simulations of STIRAP and ARP in the blockaded ensembles and of the possible experimental implementation.
[22] If the ARP pulses have the same phase, the accumulated atomic phase is also independent of N, but is equal to π.
[23] If the probability of loading at least one atom in a single ensemble is P_1, then the probability of successfully loading M ensembles after m tries with no defects is $P = 1 - (1 - P_1^M)^m$. For $\bar{N} = 5$ we have $P_1 = 1 - 0.0067$ and for $M = 100$ ensembles the probability of success after $m = 10$ tries is 0.999.
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