Addressing atoms in optical lattices with Bessel beams

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A method of synthesizing localized optical fields with zeros on a periodic lattice is analyzed. The applicability to addressing atoms trapped in optical lattices with low cross talk is discussed.

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There is much current interest in using atoms trapped in optical lattices for quantum logic devices.[1,2] One of the challenges in implementing this scheme is posed by the need to address individual atoms with near-resonant light. Current estimates of error limits for scalable quantum computing require primitive logic operations with errors as low as[3,4] $O(10^{-5}-10^{-6})$. The corresponding limitation on optical cross talk when addressing one atom in a lattice can be quantified only in the context of a particular choice of quantum gate. In atomic schemes error rates for operations that are dependent on single-photon processes tend to scale with the intensity. This implies that the intensity leakage of a logical control or state readout beam at a site adjacent to a site being addressed should not exceed $O(10^{-5})$. In this Letter I describe a novel approach to image formation that allows sites in an optical lattice to be addressed with minimal cross talk.

To emphasize the difficulty of achieving low cross talk in optical lattices, consider the following simple scaling relationships: We have a one- or two-dimensional lattice lying in the $x-y$ plane that is defined by counterpropagating beams at wavelength λ_f. Individual atoms are separated by a minimum distance of $d = \lambda_f/2$, and we wish to address them with near-resonant light of wavelength λ. The most obvious approach to doing so involves focusing a beam propagating perpendicular to the plane of the lattice (along $\hat{z}$) to a small spot. Assuming a Gaussian beam profile, the intensity distribution is $I(\rho, z) = I_0 \exp(-2\rho^2/w^2)$, where $\rho^2 = x^2 + y^2$, $w^2(z) = \omega_0^2(1 + z^2/z_R^2)$, with w_0 being the beam waist at $z = 0$ and $z_R = \pi w_0^2/\lambda$. Expressing the ratio of the intensity at a neighboring site to the on-site intensity as ϵ implies that beam waist $w_0 = [-1/(2 \ln \epsilon)]^{1/2}\lambda_f$. For $\epsilon = 10^{-5}$ this evaluates to $\tilde{w}_0 = w_0/\lambda_f = 0.21$.

We can quantify the corresponding requirement on the ratio of trapping and addressing beam wavelengths in terms of the performance of lenses with high numerical apertures (NAs). We assume that a lens system with aperture diameter D is used to focus the addressing light. If the ratio of the lens aperture to the Gaussian waist at the aperture is given by $p = D/w(z_{\text{lens}})$, then a focal plane waist of w_0 implies that $D \simeq p w_0 z_{\text{lens}}/z_R$. The ratio p must be large enough that the diffractive rings introduced by the aperture do not lead to unacceptable intensity cross talk. Since the power in the rings is spread over an entire circle, it is reasonable to allow the aperture to block a much larger fraction of the light than the desired cross-talk ratio ϵ. Using $p = 3$ yields approximately 1% transmission loss at the lens and a NA of

$$\text{NA} = \frac{[(3/2\pi\tilde{w}_0)(\lambda/\lambda_f)]}{\{1 + [(3/2\pi\tilde{w}_0)(\lambda/\lambda_f)]^2\}^{1/2}}\,. \tag{1}$$

The variation of the lens NA with $\tilde{w}_0$ is shown in Fig. 1. Note that a more conservative (larger) estimate for p would only increase the NA requirements of the lens. Available microscope objectives, as well as high-resolution optical lithography lens systems, tend to have NAs of not more than ~0.8, unless oil immersion is used, which is not compatible with atomic imaging inside vacuum chambers. This implies that the lattice light wavelength λ_f must be several times longer than the addressing wavelength λ to address single atoms with negligible leakage to neighboring sites. Thus a lattice of Rb atoms, which can be manipulated with near-resonant light with the D2 line at $\lambda = 0.78\ \mu$m, will not be individually resolvable in the context of scalable quantum logic unless $\lambda_f \gtrsim 1.5\ \mu$m.

Various solutions to the addressability problem are under study. The most direct solution is to make $\lambda_f \gg \lambda$. Arrays of widely spaced traps that use many diffractively generated beams[5] instead of lattices also fall into this category of solution. Unfortunately this is not compatible with loading from a Bose–Einstein condensate through the Mott transition,[6] although other loading schemes may still be used.[7,8] Another possibility is to keep $\lambda \sim \lambda_f$ but load the lattice so that only every few lattice sites are occupied or change the angle between the lattice beams after loading so that a longer periodicity is obtained.[9,10]

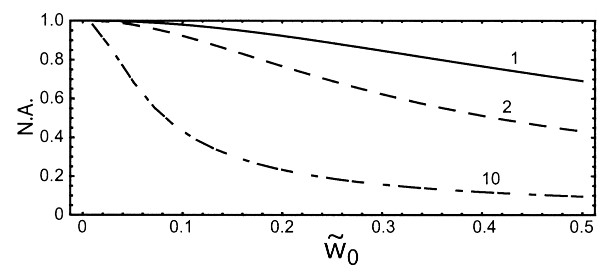

Fig. 1. Lens NA needed to focus a Gaussian beam to a waist $\tilde{w}_0$ for $\lambda_f/\lambda = 1$, 2 and 10.

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Here I analyze an alternative approach to single-atom addressing in optical lattices with $\lambda \sim \lambda_f$. The idea is to accept the fact that the light cannot be localized sufficiently well but tailor the beam profile so that the field has zeros at neighboring lattice sites. The basic geometry is shown in Fig. 2. An incident beam passes through a spatial light modulator to produce N beams, each with an amplitude and phase controlled by a single pixel of the modulator. By use of mirrors the beams are divided into four bundles that propagate in the xy plane and are focused to converge on the origin at $x = y = 0$ with lenses of focal length f. The modulator and the image synthesis region between the four lenses are assumed to be in Fourier conjugate planes (additional relay lenses not shown in the figure may be required in practice). Thus each individual beam results in a plane wave that illuminates the origin from a different angle. Each plane wave is polarized along $\hat{z}$ with amplitude $A_j \exp[i(-\mathbf{k}_j \cdot \boldsymbol{\rho} - \omega t + \chi_j)]$ + c.c., where $\mathbf{k}_j = k(\cos\phi_j \hat{x} + \sin\phi_j \hat{y})$, $\boldsymbol{\rho} = x\hat{x} + y\hat{y}$, $k = 2\pi/\lambda$, ϕ_j is the azimuthal angle of wave j, and A_j and χ_j are adjustable amplitudes and phases, respectively. In the limit when $N \to \infty$ and the beam amplitudes and phases are all equal the field amplitude generated on axis is the zeroth-order Bessel beam $J_0(k\rho)$.

When the field is synthesized from a finite number N of plane waves it is quasi-periodic in space, and rings of secondary interference maxima occur, as can be seen in Fig. 3. The diameter of the secondary rings can be estimated from $\Delta k d_{\text{ring}} \simeq 2\pi$. Using $\Delta k = 2k/(N/2)$ yields $d_{\text{ring}} \simeq N\lambda/4$, whereas the actual diameter found numerically is 25% larger. From the shift theorem of Fourier analysis it is straightforward to scan the spot location over a distance of up to d_{ring} by adding appropriate phase offsets to the incident beams, as shown in Fig. 3. It is possible in this way to address hundreds of atoms with a multipixel modulator, without mechanical motion of the optical system. This approach to synthesis and scanning of localized optical fields also has application in atomic lithography.[11]

A Bessel beam written with $\lambda = 0.78\ \mu$m has a central lobe with a $1/e^2$ intensity radius of 0.22 μm. However, the closely spaced secondary maxima of the Bessel function result in cross talk on nearby lattice sites. I describe now a method of synthesizing a beam with zeros at regularly spaced sites on a one-dimensional lattice. An arbitrary solution of the two-dimensional Helmholtz equation can be written as a Fourier–Bessel series:

$$A(\rho, \theta) = J_0(k\rho) + \sum_{n=1}^{\infty} a_n J_n(k\rho)\exp(in\theta), \tag{2}$$

where ρ and θ are radial and angular coordinates, respectively, and the a_n are complex coefficients to be determined. I set the zeroth-order coefficient to unity, corresponding to a normalized field with unit amplitude at the origin. Any field of the form of Eq. (2) can be synthesized with an amplitude and phase modulator in the geometry of Fig. 2. To make this explicit, note that the Fourier transform of Eq. (2) is

$$\begin{aligned}\tilde{A}(q, \phi) &= \int_0^{\infty} \mathrm{d}\rho\, \rho \int_0^{2\pi} \mathrm{d}\theta A(\rho, \theta)\exp[i\rho q \cos(\theta - \phi)] \\ &= \frac{2\pi}{k}\, \delta(q - k)\left[1 + \sum_{n=1}^{\infty} a_n i^n \exp(in\phi)\right]. \end{aligned} \tag{3}$$

Thus a ring of converging plane waves with wave number k and complex amplitudes given by the term in brackets in Eq. (3) provides the desired field. The role of the lenses shown in Fig. 2 is to tilt the wave passing through each spatial light modulator pixel toward the center of the image region, but the lenses do not tightly focus each wave. Outside the central spot, destructive interference leads to a low background light level. From the point of view of Fourier optics, the focusing of a beam results from interference of the constituent plane-wave components, taking into account relative phase shifts caused by propagation. The usefulness of the approach to image synthesis presented here is that the amplitudes and phases of each of the plane-wave components can be directly controlled.

A field consisting of a finite number of terms in Eq. (2) that is useful for addressing a one-dimensional

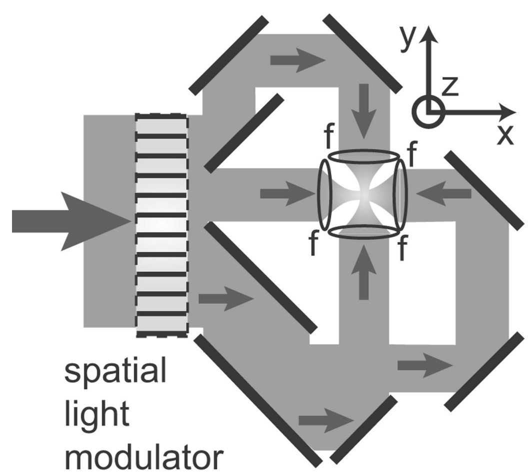

Fig. 2. Optical system for producing a Bessel beam with a one-dimensional spatial light modulator that has N pixels. The additional beams of wavelength λ_f necessary for creating an optical lattice can in principle be combined with the imaging beams by use of dichroic mirrors not shown in the figure.

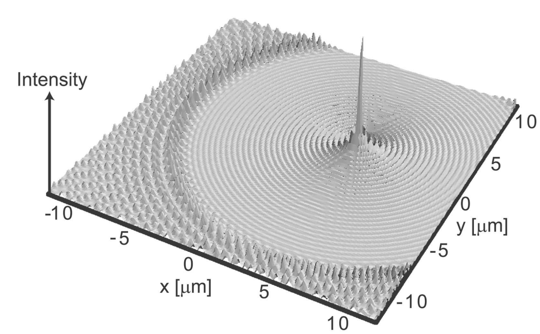

Fig. 3. Intensity distribution calculated with $N = 100$ and $\lambda = 0.78\ \mu$m. The central lobe was translated by $\Delta x = 4\ \mu$m and $\Delta y = 2\ \mu$m by adding phase offsets; see the text for details.

Table 1. Bessel Coefficients and Resulting Maximum Cross Talk at Site $m_{\max}$ away from the Origin[a]

$M =$	1	2	3	4	5	6
a_2	0.675	0.715	0.725	0.728	0.730	0.731
a_4		−0.118	−0.150	−0.163	−0.170	−0.174
a_6			0.0406	0.0616	0.0736	0.0814
a_8				−0.0169	−0.0302	−0.0401
a_{10}					0.00778	0.01622
a_{12}						−0.003857
$\max\|A\|^2$	3.0×10^{-3}	6.1×10^{-4}	1.9×10^{-4}	7.1×10^{-5}	3.6×10^{-5}	3.3×10^{-5}
$m_{\max}$	4	8	11	14	19	28
14-bit $\|A\|^2$	2.9×10^{-3}	6.1×10^{-4}	2.0×10^{-4}	9.3×10^{-5}	7.9×10^{-5}	7.7×10^{-5}

[a]Calculated with $\lambda = 0.78\ \mu$m and $\lambda_f = 0.8\ \mu$m. The last row shows the maximum error in the first 50 neighboring sites using Eq. (3) with $N = 256$.

lattice along $\hat{x}$ will have $A(\rho, 0) = A(\rho, \pi)$, which implies that $a_n = 0$ for n odd. Requiring that A vanish at the M lattice points specified by $\rho_m = m\lambda_f/2$, $\theta_m = 0$ for $m = 1, \ldots, M$ and limiting the sum in Eq. (2) to M term results in M linearly independent equations $J_0(k\rho_m) + \sum_{n=1}^{M} a_{2n} J_{2n}(k\rho_m) = 0$. These equations are easily solved for the coefficients a_{2n}. The coefficients decrease rapidly with the Bessel order, provided that λ_f is not too much less than λ. The series coefficients and maximum intensity cross talk at any site not being addressed are listed in Table 1. We see that $M = 6$ is sufficient to ensure a cross talk of a few times 10^{-5}. In an actual implementation there are several limiting factors to consider, including the number of spatial light modulator pixels and the amplitude and phase resolution of each one. The last row in Table 1 shows that the cross talk is up to several times higher than the theoretical value if we assume that the field is synthesized from 256 beams equally spaced azimuthally, with 14-bit resolution in amplitude and phase modulation for each beam. Additional calculations with $\lambda_f = 1.0\ \mu$m yield cross-talk levels that are roughly ten times lower than the example in Table 1. We expect a primary experimental challenge to be posed by static phase errors caused by imperfect alignment and optical component tolerances. In principle, these can be corrected for use of the spatial light modulator. For example, a detector set to monitor the fluorescence from an atom at the site being addressed would provide a signal that would increase with the peak amplitude of the field and hence indicate when the phases on each pixel have been adjusted correctly.

In summary, I have described a novel method of addressing atoms in periodic one-dimensional lattices with low cross talk. The extension of this approach to a two-dimensional lattice is complicated by the fact that a two-dimensional lattice has angle-dependent interatomic spacings. The Fourier–Bessel expansion in Eq. (2) can still be used, but the coefficients tend to grow rapidly with n. Generalizations that are suitable for a two-dimensional geometry are currently under investigation.

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