Modification of pattern formation in doubly resonant second-harmonic generation by competing parametric oscillation

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We analyze pattern formation in doubly resonant intracavity second-harmonic generation in the presence of competing nondegenerate parametric downconversion. We show that for positive cavity detuning of the fundamental frequency the threshold for parametric oscillation is lower than that of transverse, pattern forming instabilities. The parametric oscillation strongly modifies the pattern dynamics found previously in a simplified analysis that neglects parametric instability [Phys. Rev. E **56,** 4803 (1997)]. Stationary and dynamic patterns in the presence of parametric oscillation are found numerically. © 2000 Optical Society of America

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$\chi^{(2)}$ nonlinear frequency-conversion processes have attracted considerable interest for use in studies of instabilities and transverse structures.[1–6] The combination of quadratic nonlinearity and diffraction in the presence of cavity feedback leads to a rich variety of pattern formation both in optical parametric oscillators[1–3] (OPO's) and in second-harmonic generation (SHG).[4–6] Furthermore, interesting prospects for generating spatial structures with quantum-mechanical properties have been proposed.[7]

In the description of efficient SHG in a cavity it is important to include the possibility for the generated second-harmonic field to decay through a nondegenerate parametric process. This competing parametric process can influence all dynamics substantially, thus making a pure SHG analysis, with the parametric process ignored, insufficient. The importance of the parametric process in relation to generation of nonclassical light in SHG was elucidated by Marte[8] and verified experimentally by Schiller and co-workers.[9] Furthermore, parametric quenching of the temporal self-pulsing instability that exists in a pure SHG system has been discussed.[10] Pattern-formation dynamics can also change as a result of the competing parametric process, as discussed in Ref. 6 for the singly resonant configuration, in which only the fundamental field is resonated in the cavity. It was shown there that large parameter regions exist where the parametric process is the instability with the lowest threshold, which results in different transverse structures from the square and hexagonal patterns found in the pure SHG system.

Here we investigate the importance of the competing parametric process in doubly resonant SHG in which both the fundamental and the second-harmonic fields are resonated in a cavity. The threshold for generation of the parametric fields is minimized when they oscillate at frequencies close to a cavity resonance. All four fields are therefore quasi-resonant with the cavity. We performed a linear stability analysis to obtain the threshold for onset of the parametric process, and here we compare the threshold with the other instabilities that exist in the pure SHG system. Nonlinear pattern formation above the instability threshold is investigated numerically, and the modifications that are due to the presence of the parametric process are pointed out.

One can derive the coupled mean-field equations for the amplitudes of the four cavity fields by starting from paraxial propagation equations. In the case of ideal phase matching of both nonlinear processes, we obtain the normalized set

$$\partial A_1/\partial t = (-1 + i\Delta_1)A_1 + iA_1{}^* A_2 + i\nabla_\perp{}^2 A_1 + E\,, \tag{1a}$$

$$\partial A_2/\partial t = (-\gamma + i\Delta_2)A_2 + iA_1{}^2 + 2iA_+A_- + \frac{i}{2}\nabla_\perp{}^2 A_2\,, \tag{1b}$$

$$\partial A_\pm/\partial t = (-1 + i\Delta_\pm)A_\pm + iA_\mp{}^* A_2 + i\nabla_\perp{}^2 A_\pm\,, \tag{1c}$$

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where A_1, A_2, and $A_\pm$ are the amplitudes of the fundamental, the second-harmonic, and the parametric fields, respectively, E is the external pump field at the fundamental frequency, $\gamma = \gamma_2/\gamma_1$ is the ratio between the loss rates of the second harmonic and the fundamental, and the cavity-detuning parameters Δ_1, Δ_2, and $\Delta_\pm$ measure the difference between the optical frequency and the nearest cavity resonance frequency, normalized to γ_1. To achieve resonance of the fundamental and the second harmonic simultaneously we need to tune the two fields independently; thus Δ_1 and Δ_2 are independent parameters. The parametric fields, however, are emitted close to a cavity resonance positioned an integer multiple of the cavity's free spectral range away from the fundamental resonance, and energy conservation implies a relation between the detunings: $\Delta_+ + \Delta_- = 2\Delta_1$.[6] Furthermore, we consider the symmetric case in which the parametric fields are detuned equally with respect to the nearest cavity resonance, and thus $\Delta_+ = \Delta_- = \Delta_1$. Finally, the parametric fields are assumed to experience the same cavity round-trip losses as the fundamental, which is a good assumption because their frequencies are close to the fundamental frequency as a result of phase-matching bandwidth restrictions in the OPO process.

The starting point for the analysis is to find the homogeneous solutions $A_1{}^0$, $A_2{}^0$, and $A_\pm{}^0$. The fundamental and the second harmonic are obtained by solution of a cubic equation,[4] whereas we assume that $A_+{}^0 = A_-{}^0 = 0$ for the parametric fields; i.e., the stability analysis determines the parametric oscillation threshold. We introduce transverse perturbations of the homogeneous solutions of the form $A_j = A_j{}^0 + a_j \exp(\lambda t + i\mathbf{k}_\perp \cdot \mathbf{r}) + b_j \exp(\lambda^* t - i\mathbf{k}_\perp \cdot \mathbf{r})$ and $A_\pm = a_\pm \exp(\tilde{\lambda} t + i\tilde{\mathbf{k}}_\perp \cdot \mathbf{r}) + b_\pm \exp(\tilde{\lambda}^* t - i\tilde{\mathbf{k}}_\perp \cdot \mathbf{r})$, where $j = 1, 2$ and $\mathbf{r} = (x, y)^T$ is the coordinate vector in the transverse plane. Substituting into Eqs. (1) and linearizing in the perturbations lead to two decoupled quartic characteristic polynomials in the eigenvalues λ and $\tilde{\lambda}$, respectively. Focusing only on the equation that involves λ corresponds to a pure SHG system in which the competing parametric process is not excited, as was analyzed in detail in Ref. 4, in which the presence of bistability and self-pulsing in addition to both stationary and dynamic pattern-forming transverse instabilities was demonstrated.

The characteristic equation that involves $\tilde{\lambda}$ can be written as

$$[\tilde{\lambda}^2 + 2\tilde{\lambda} + 1 + (\tilde{k}_\perp{}^2 - \Delta_1)^2 - |A_2{}^0|^2]^2 = 0. \quad (2)$$

Setting the real part of eigenvalue $\tilde{\lambda}$ equal to zero, we obtain the threshold for onset of the parametric process:

$$|A_{1,p}{}^0|^2 = (\gamma^2 + \Delta_2{}^2)^{1/2}[1 + (\tilde{k}_\perp{}^2 - \Delta_1)^2]^{1/2}. \quad (3)$$

Depending on the sign of the fundamental detuning Δ_1, the parametric fields are emitted either on axis, $\tilde{k}_\perp = 0\,(\Delta_1 < 0)$, or off axis, $\tilde{k}_\perp = \sqrt{\Delta_1}\,(\Delta_1 > 0)$, in the same way as in the ordinary OPO.[1]

The competing parametric process is of considerable importance in the description of pattern formation in doubly resonant SHG. For positive values of fundamental detuning Δ_1, the parametric threshold is generally lower than the other instabilities, which is due to the threshold lowering associated with the off-axis emission,[1] as shown, for example, in Fig. 1 for $\Delta_1 = 2$. For negative detuning of the fundamental, however, it is possible to obtain SHG instabilities below the parametric threshold, as shown in Fig. 2 for $\Delta_1 = -2$. Here we observe that the stationary transverse SHG instability is lowest for large negative values of Δ_2. Furthermore, for Δ_2 close to zero there are regions where both the parametric threshold and the temporal self-pulsing threshold are lowest, whereas for large positive Δ_2 the oscillatory transverse SHG instability appears first. In both figures, limit points from bistability of the homogeneous solutions lie outside the plotted range.

The qualitatively different regimes $\Delta_1 < 0$ and $\Delta_1 > 0$ are investigated through numerical analysis. Details of the numerical method can be found in Ref. 6. In general, for Δ_1 positive, the parametric process dominates the behavior, so spatial structures seen in a pure SHG system will be suppressed. For instance, for $\Delta_1 = 2$ and $\Delta_2 = -2$ we observe a

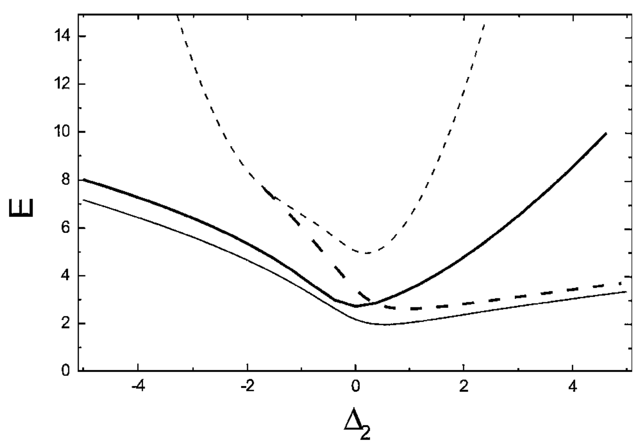

Fig. 1. Pump amplitude E for the instability thresholds as a function of second-harmonic detuning Δ_2 and with $\gamma = 0.6$ and $\Delta_1 = 2$. Thin solid curve, parametric process; thick solid curve, stationary transverse instability; thin dashed curve, self-pulsing instability; thick dashed curve, oscillatory transverse instability.

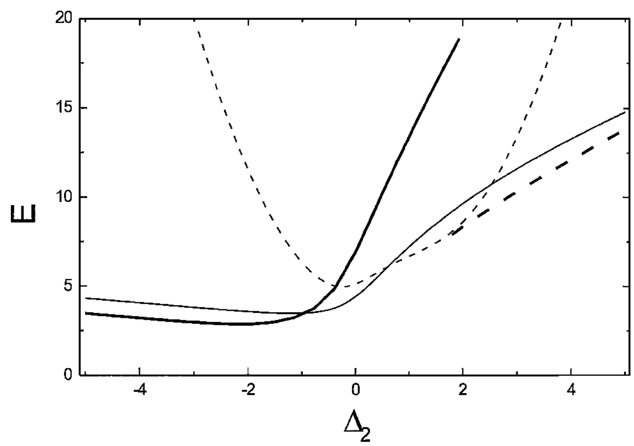

Fig. 2. Bifurcation diagram for $\Delta_1 = -2$ and otherwise the same parameters and notation as for Fig. 1.

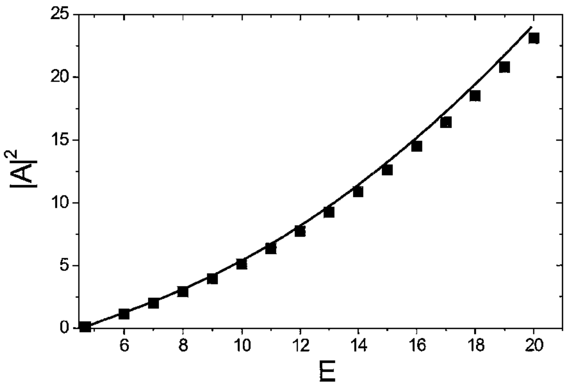

Fig. 3. Comparison between analytical (solid curve) and numerical (filled squares) solutions for $\Delta_1 = 2$, $\Delta_2 = -2$, $\gamma = 0.6$.

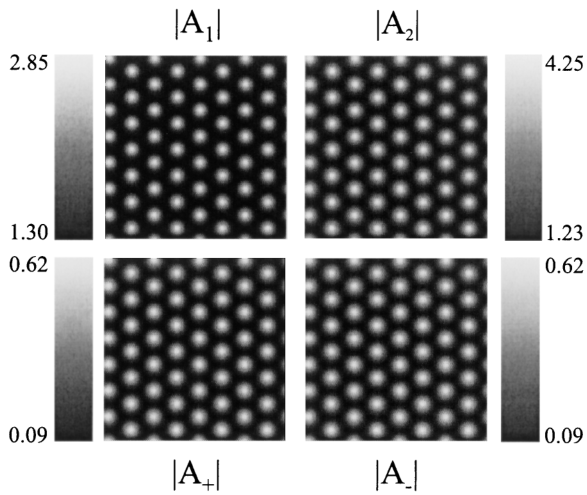

Fig. 4. Near-field intensity distributions for $\Delta_1 = -2$, $\Delta_2 = -1$, $\gamma = 0.6$, $E = 3.4$. The window size is 36×36.

clear hexagonal symmetry in the pure SHG system, whereas, with the parametric fields included, homogeneous solutions are found. As expected in this case of positive detuning, the parametric fields are emitted off axis, corresponding to plane-wave solutions of the form $A_\pm = A\exp(\pm i\tilde{\mathbf{k}}_\perp \cdot \mathbf{r})$. Note the difference compared with the ordinary nondegenerate OPO, for which time-oscillating traveling waves of the form $A_\pm = A\exp[\pm i(\tilde{\mathbf{k}}_\perp \cdot \mathbf{r} - \tilde{\omega}_c t)]$ with a frequency $\tilde{\omega}_c$ are found.[3] However, in our system the parametric instability appears to be stationary, and thus $\tilde{\omega}_c = 0$. The homogeneous solutions are highly robust and do not seem to become unstable even when the system is driven far above threshold. Thus in this case the parametric process leads to a complete suppression of the pattern-formation dynamics otherwise observed. It is possible to derive an exact analytical solution for amplitude A of the parametric solution that can be used as a test of the numerical analysis. The explicit expression is lengthy and is not given here. In Fig. 3 the analytical solution is plotted together with the numerical solution for $\Delta_1 = 2$, $\Delta_2 = -2$, and $\gamma = 0.6$, and very good agreement is obtained.

For Δ_1 negative, instabilities seen in pure SHG can prevail in the presence of the parametric process. As can be seen from Fig. 2, e.g., for $\Delta_1 = -2$ and $\Delta_2 = -1$, the pure SHG stationary transverse instability is lowest, leading to the formation of hexagons. When E is increased above the parametric threshold, the pattern structure can survive, and hexagons in all four fields are obtained, as shown in Fig. 4. For $\Delta_1 = -2$ and $\Delta_2 = 1$, purely temporal, self-pulsing instability can be addressed. Above the parametric threshold, self-pulsing of all four fields is obtained, unlike in the on-resonance case $\Delta_1 = \Delta_2 = 0$ studied in Ref. 10, in which the presence of the parametric fields was found to inhibit self-pulsing. Finally, oscillatory transverse instability also can be seen, e.g., for $\Delta_1 = -2$ and $\Delta_2 = 2$, where traveling roll patterns are found.

In conclusion, we have described pattern formation in doubly resonant SHG with the inclusion of a competing parametric process. Through stability analysis, two fundamentally different parameter regions were identified that depend on the sign of Δ_1. For Δ_1 positive, the parametric instability dominates and influences the dynamics decisively, whereas the instabilities seen in a pure SHG system can be addressed for Δ_1 negative.

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References

1. G.-L. Oppo, M. Brambilla, and L. A. Lugiato, Phys. Rev. A **49,** 2028 (1994).
2. K. Staliunas, J. Mod. Opt. **42,** 1261 (1995); G. J. de Valcárcel, K. Staliunas, E. Roldán, and V. J. Sánchez-Morcillo, Phys. Rev. A **54,** 1609 (1996).
3. S. Longhi, Phys. Rev. A **53,** 4488 (1996).
4. C. Etrich, U. Peschel, and F. Lederer, Phys. Rev. E **56,** 4803 (1997).
5. S. Longhi, Opt. Lett. **23,** 346 (1998).
6. P. Lodahl and M. Saffman, Phys. Rev. A **60,** 3251 (1999).
7. L. A. Lugiato and A. Gatti, Phys. Rev. Lett. **70,** 3868 (1993); A. Gatti, H. Wiedemann, L. A. Lugiato, I. Marzoli, G.-L. Oppo, and S. M. Barnett, Phys. Rev. A **56,** 877 (1997).
8. M. A. M. Marte, Phys. Rev. Lett. **74,** 4815 (1995).
9. S. Schiller and R. L. Byer, J. Opt. Soc. Am. B **10,** 1696 (1993); A. G. White, P. K. Lam, M. S. Taubman, M. A. M. Marte, S. Schiller, D. E. McClelland, and H.-A. Bachor, Phys. Rev. A **55,** 4511 (1997).
10. M. A. M. Marte, Phys. Rev. A **49,** R3166 (1994).