Fourier control of pattern formation in an interferometric feedback configuration

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Abstract

We present a control method for the minimally invasive manipulation of pattern states occurring in feedback systems with an optical nonlinearity. An interferometric feedback configuration is used to control spontaneously formed patterns by the method of Fourier filtering. This method is minimally invasive since the pattern forming feedback arm remains unchanged, while the energy removed by the filter or the control signal added to the system by the second control feedback arm tends to a small level when the desired pattern is obtained. Here, we present experimental results for a photorefractive two-arm feedback system, including switching from hexagons to rolls or squares by positive (in-phase) control and manipulation of the hexagon orientation by negative (out-of-phase) control. A comparison with results of a numerical simulation based on a thin saturable Kerr-slice model is performed. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

A large number of physical, chemical, or biological systems that are excited beyond an instability threshold are known to lead to the spontaneous formation of periodic spatial structures [1,2]. Nonlinear optical systems are well-suited for the investigation of these structures due to the easy access to control parameters. Moreover, pattern formation in optics offers a wide range of possible technological applications, for example in information processing [3].

A simple configuration using a nonlinear optical material and a feedback mirror for creating the counterpropagating beam is sufficient for observing transverse structures. Single feedback systems containing liquid crystals or liquid crystal light valves [4–6], atomic vapours [7–9], organic films [10], and photorefractives [11,12] show similar behaviour when excited beyond the instability threshold. The predominant structure the systems select is hexagonal, but even squares, squeezed hexagonal or roll patterns [13–15] can be excited in a photorefractive single feedback system. For fixed parameters, the nonlinear optical system selects only one spatial pattern, other patterns are suppressed and may not be accessible to the observer. So far, theoretical investigations of the

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photorefractive system are limited to the linear stability analysis for the feedback-mirror case [12], as well as a recently presented nonlinear stability analysis of hexagonal structures [16].

Optical pattern control represents a fast growing field of research and has attracted considerable interest [17]. Especially in the realm of technological applications, it is of interest to control the pattern the system selects, i.e. to access the maximum number of different existing patterns in a special parameter region. In this context, the stabilization and manipulation of otherwise unstable pattern states is currently of high interest. A control method based on spatial filtering in the Fourier spectrum of the pattern was proposed theoretically [18,19], and soon afterwards demonstrated experimentally [20–23]. Spectral control techniques are extremely attractive for optical systems since the Fourier space is accessible in

real-time by using a single lens. The control method originally proposed in Ref. [18] has the important property that it is basically noninvasive. When the desired state is reached, the control signal declines to a very small level as in conventional electronic control schemes and otherwise unstable states of the underlying system are stabilized. Here, we present results for minimally-invasive control of a photorefractive feedback system by using a Michelson-interferometer feedback scheme.

As shown in Fig. 1, we investigate a pattern forming system with a two-arm interferometric feedback. The pattern arm of the interferometer provides the majority of the energy in the feedback loop and serves as the unperturbed pattern forming system. The other arm, which we call the control arm, filters a small portion of energy in order to control and select the patterns in the pattern arm. We distinguish
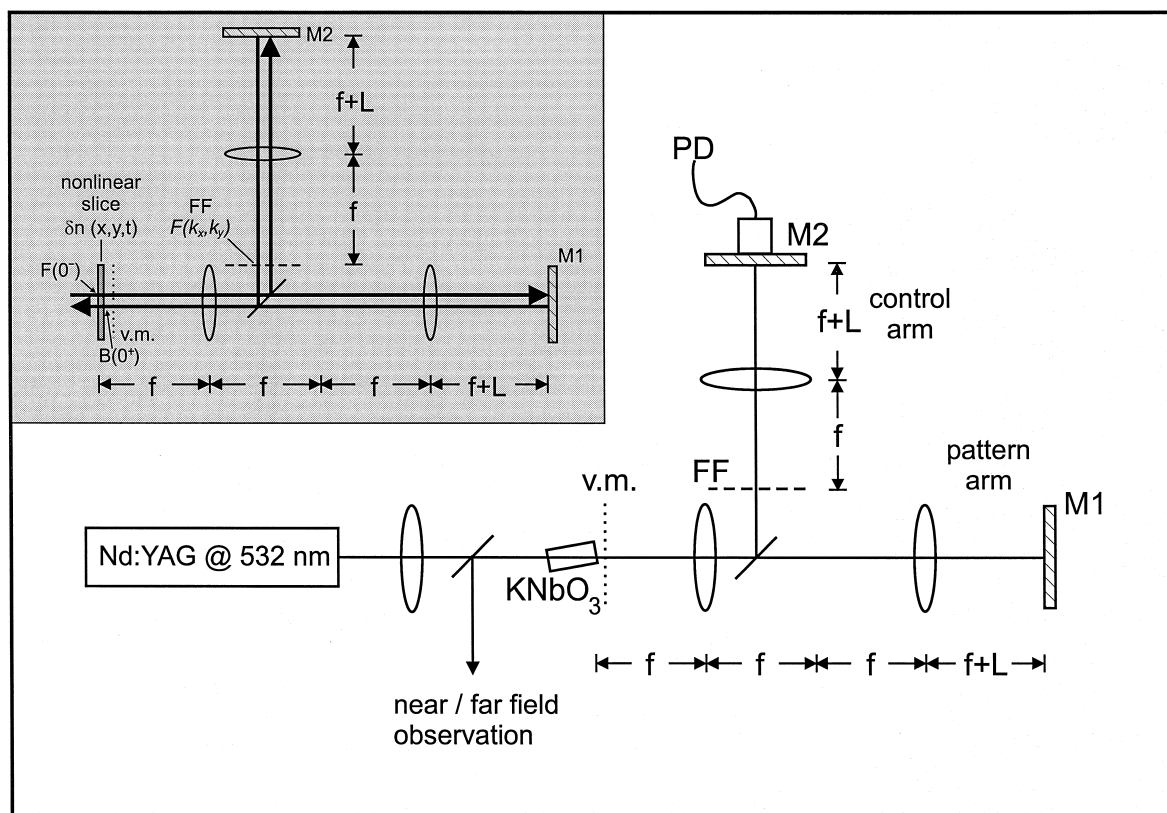

Fig. 1. Experimental set-up. M = mirror, v.m. = virtual mirror, PD = piezo driver, FF = Fourier filter, L = propagation length. Grey-shaded inset: Theoretical model with nonlinear slice and forward (**F**) and backward beam (**B**)

between the terms positive control, where the control signal is fed back constructively (and thereby enhancing a desired pattern) and negative control. In the latter case, unwanted components are suppressed by destructive feedback of the control signal which nearly vanishes when achieving the desired state. In Fig. 2, examples for the control signals for positive and negative control are shown. In the case of the positive control, the control signal (i.e. the energy that is fed back into the system) tends to a certain steady-state level, whereas negative control implies that the control signal declines to a very small level in the desired pattern state. In this context, the term 'minimally' means for the positive control that the control signal should be as small as possible at the very first stage of control and in the asymptotic steady state. By applying this positive control technique, we are able to excite rolls and squares in a parameter region, where only a hexagonal pattern is experimentally available without any control. For the negative control method, the minimally-invasive property also implies the minimization for the initial stage of the control process as well as the fact that the control signal declines to a very small but non-zero level. This negative control method is sufficient to rotate the spatial orientation of the hexagonal pattern. It is worth mentioning here that we do not restrict ourselves to a 'noninvasive' stabilization of otherwise unstable states as proposed in Refs. [18,19]. We perform a minimally-invasive manipulation and control of patterns, and demonstrate the selection of different pattern states. Our aim is to manipulate the system in a way that other patterns become stable in parameter space regions, where, without any control, the hexagonal pattern is the only experimentally obtainable solution.
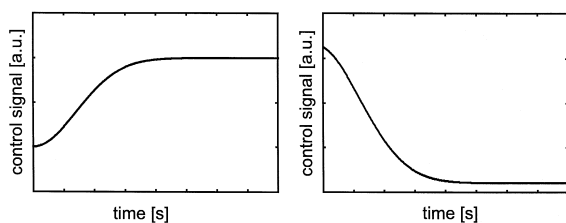

Fig. 2. Examples for control signals for positive control (left) and negative control (right)

2. Experiment

2.1. Experimental set-up

The experimental set-up is depicted in Fig. 1. The beam derived from a frequency-doubled Nd:YAG-laser operating at a wavelength of $\lambda = 532$ nm was focused onto the exit face of a photorefractive $KNbO_3$:Fe-crystal measuring $l = 5.2$ mm along the c-axis. Polarization was chosen to be along the crystal's a-axis to exploit the large r_{13}-component of the electrooptic tensor. The propagation direction of the light was nearly parallel to the c-axis of the crystal which was oriented such that the incoming beam was depleted when passing the crystal. The power incident on the crystal was $P = 10$ mW and the Gaussian diameter of the beam was measured to be $d = 320$ μm. Feedback is provided by an interferometric set-up of Michelson-type, separating the pattern from the control arm. Both feedback arms are basically identical 4f–4f imaging systems, containing lenses of focal lengths $f = 100$ mm and mirrors M1 and M2, respectively. A beam splitter right after the first lens in the feedback arm provides access to the control arm, where manipulation in Fourier space is performed. Amplitude masks of different shape can be inserted in the Fourier plane that is created between the two lenses.

Let us now assume that only the pattern arm operates, i.e. the control arm is closed. The 4f–4f imaging system with a diffraction length L creates a virtual mirror at a distance L behind the crystal, incorporting the fundamental advantage that negative propagation lengths can be achieved [24] and that the Fourier spectrum of the pattern is automatically created between the two lenses of the feedback arm. The lenses were adjusted to give a virtual mirror (v.m.) position of $L = 1$ mm behind the crystal in order to create a hexagonal pattern with a large aspect ratio.

Transverse instability in this geometry is known to be due to reflection gratings [12,25] with a wave number of $k_g = 2k_0 n_0$, where k_0 is the wave number of the incoming beam, and n_0 the linear refractive index of the crystal. $KNbO_3$ is known to be a purely diffusion-dominated material exhibiting energy-only coupling between the counterpropagating

beams. Closer analysis of this simple feedback system is presented elsewhere [12,13,15] and is not the main topic of this paper. It is noteworthy that in certain parameter space regions, especially for situations when the virtual mirror is inside the crystal, a number of different pattern states exist, as square, squeezed hexagonal [13] or even rectangular or parallelogram-like patterns [26]. Nevertheless, the structure the system selects for the given virtual mirror position of $L = 1$ mm is purely hexagonal, as depicted in Fig. 3, top left picture.

2.2. Control method

This Michelson-like control method represents an extension and an improvement of our previously published ring control [21] or linear control scheme

[22], since the pattern forming and the control arm are completely separated from each other. The propagation lengths of both arms can be adjusted independently and the relative phase of the two arms can be monitored easily at the appropriate exit of the beam splitter. A piezo-driven mirror is used for controlled adjustment of the relative phase of both arms, i.e. for achieving negative (relative phase $\varphi = \pi$) or positive (relative phase $\varphi = 0$) control. This scheme allows us (by means of an attenuator in the control arm) to adjust continuously the control strength in order to minimize it. The reflectivity ratio of both feedback arms was $R_1/R_2 = 1.5$ with $R_1 = 15.8\%$ and $R_2 = 10.5\%$. R_1 and R_2 are defined as the intensity reflectivities (including both passes through the beam splitter, lens system and mirror reflectivi-
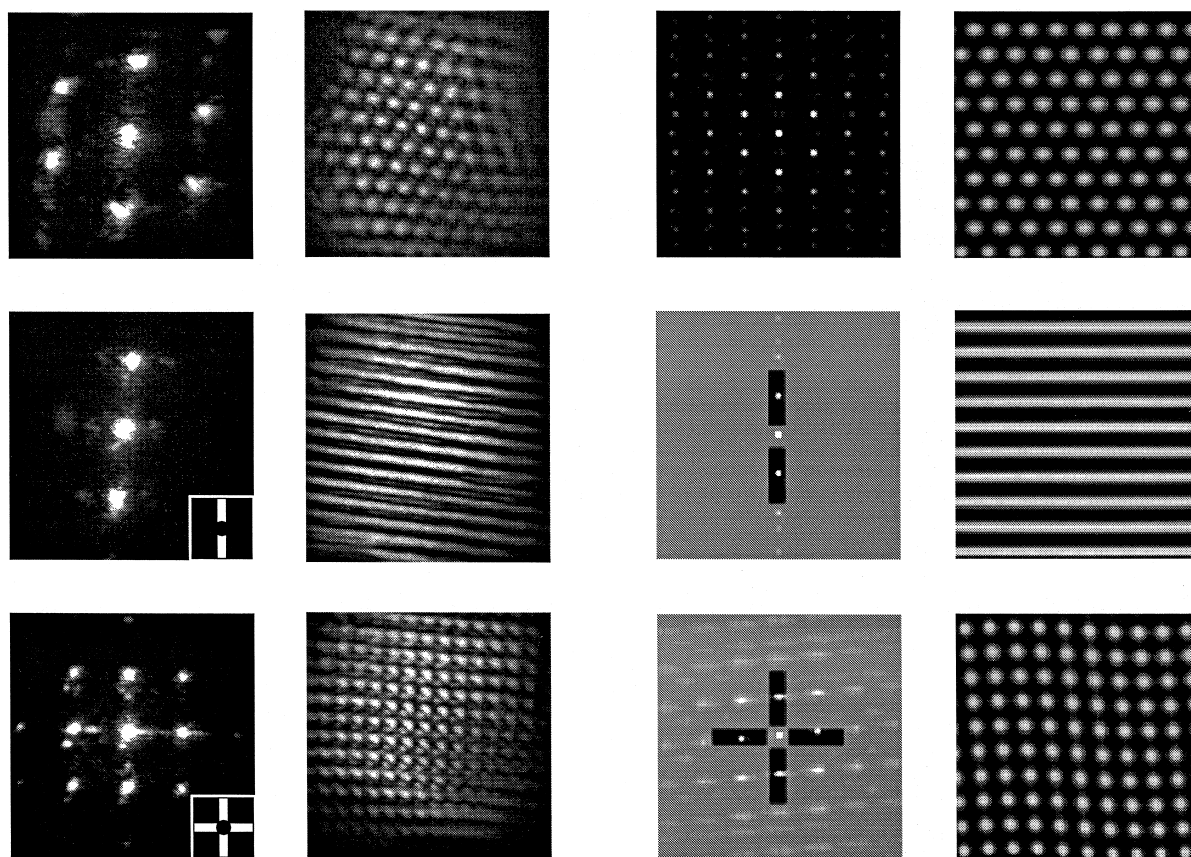

Fig. 3. Experimental (left) and theoretical results derived from a numerical simulation (right) for in-phase control. Top row: Hexagonal pattern appearing without control in the far and near field; middle and lower row: Results for the preparation of a roll and a square pattern with filters as indicated in the insets, grey-shaded in the simulations.

ties) of the pattern arm and the control arm, respectively. The propagation lengths of both arms were adjusted to be $L = 1$ mm leading to identical transverse scales of the hexagonal patterns created by both arms individually. Control is performed in the Fourier plane of the control arm, where Fourier filters of appropriate shapes can be inserted. The Fourier filters we use are binary amplitude masks with different shapes: a slit filter for positive control of rolls, a cross filter for positive control of squares and a three-fold slit filter for negative control of hexagons.

2.3. Results

Examples for the experimental positive control of a roll and a square pattern are depicted on the left of Fig. 3. The central spot was always blocked in our experiments in order to avoid undesired effects from two-beam interference which may lead to different complicated pattern formation effects [27–29]. The system adjusts to the presented symmetry of the Fourier filter (as shown in the insets) and selects the corresponding solution. Clear and rapid switching behaviour on time scales comparable to the time

constant of the crystal of less than 1 s were observed. When the control path was closed, the predominant hexagonal structure always reappeared. Note that in the case of the positive control, the control signal does not vanish. Therefore, it is not a stabilization of unstable states, but an improvement compared to previous approaches that used a single feedback arm [20,30]. The control signal (defined as the control intensity with respect to the intensity in the original pattern forming arm without control) was just 4.7%, but it was sufficient to control the behaviour of the whole system. This value was the best obtainable in our experiment, a further minimization may be possible by optimizing the experimental set-up (wavefront alignment, relative phase of both arms etc.). The control strength in the notation given in Refs. [18,19] can be calculated to be $s = 0.3$ for roll and square pattern control. In both cases, the power absorbed by the filter declines to a small level.

As an example of negative control, we manipulated the orientation of the hexagonal pattern by using a three-fold slit mask, where the slits were tilted by 60° relative to each other (see Fig. 4). Here, the control signal (the power reinjected into the
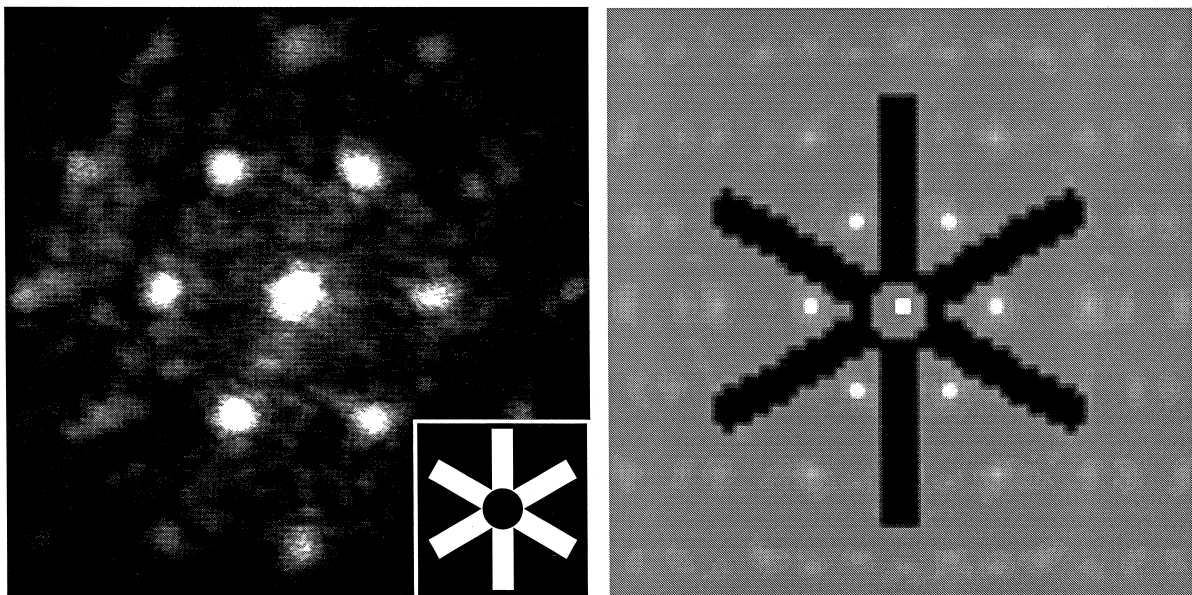

Fig. 4. Rotation of the hexagon position by out-of-phase control. Experimentally obtained pattern (left, with filter in the inset) and result from a numerical simulation, filter grey-shaded, on the right

system) tends to a very small level as suggested and calculated in Ref. [18] with the difference that, in this case, our aim is not to stabilize unstable states as the 'negative control' defined in Ref. [18] requires. Here, we manipulate a given stable pattern regarding its orientation. Note that the system remained in the chosen position when the control arm was closed. It is noteworthy that the hexagon position is freely rotatable so that by turning the filter, an arbitrary position in space can be obtained.

3. Comparison with numerical simulations

For comparison with theory, we performed a numerical simulation using the same two-dimensional feedback (see inset of Fig. 1). A numerical analysis of the photorefractive two-beam coupling equations in the presence of reflection gratings and diffraction is computationally expensive. Results to date are limited to a single transverse dimension [31,32]. A nonlinear stability analysis has already been performed [16], showing the stability of hexagons for small positive values of L, but two-dimensional analysis for other spatial symmetries is still missing. Therefore we use a simplified generic model based on a transmission grating mediated interaction in a thin slice of a sluggish medium with cubic nonlinearity [4,20]. With this model, the basic features of pattern formation can be studied. Experimental results for Fourier filtering in the pattern arm in the photorefractive system were shown to correspond to numerical simulations using the model described above in Ref. [20].

In the simplified model the nonlinear refractive index δn satisfies the relaxation equation

$$\left(\tau \frac{\partial}{\partial t} + 1\right)\delta n = \mathscr{D}\nabla_\perp^2 \delta n + \gamma \frac{|F|^2 + |B|^2}{1 + |F|^2 + |B|^2}, \quad (1)$$

where τ is the relaxation time of the medium, $\mathscr{D}$ is the diffusion coefficient, and F and B are the amplitudes of the forward and backward propagating electric field. The influence of the nonlinearity on the field amplitude is given by $F(x, y, 0^+) = F(x, y, 0^-)e^{i\delta n}$ and $B(x, y, 0^+) = B(x, y, 0^-)e^{i\delta n}$.

Free space diffraction and Fourier filtering is described by the terms

$$\frac{\partial F(x,y,z)}{\partial z} = \frac{i}{2k}\nabla_\perp^2 F(x,y,z), \quad (2)$$

$$\tilde{B}(k_x, k_y) = \tilde{F}(k_x, k_y)\mathscr{F}(k_x, k_y), \quad (3)$$

$$-\frac{\partial B(x,y,z)}{\partial z} = \frac{i}{2k}\nabla_\perp^2 B(x,y,z), \quad (4)$$

where tildes denote the Fourier transformed functions. $\mathscr{F}(k_x, k_y)$ represents the transmission function of the Fourier filter, where blocking is represented by a zero value, and 1 otherwise. The numerical simulation was performed using the same values as given in the experiment ($L = 1$ mm, $\lambda = 532$ nm, and $R_1/R_2 = 1.5$). The simulation started from an initial hexagonal pattern (as depicted on the top right of Fig. 3), which is the solution for this system without using a Fourier mask. When a spatial filter was inserted (grey-shaded region) with a certain geometry, the system adjusted to the situation given, and a roll or a square pattern could be excited. Comparing the experimental on the left and the numerical results on the right side of Fig. 3 yields a complete qualitative agreement, despite the differences in the nonlinearities used for experiment and numerical simulation. All numerical simulations were performed using a 64×64 grid and checked with a 128×128 grid. In addition, simulations were run long enough to ensure a steady-state solution.

The slight deviation from the ideal square solution in the numerical simulation may be attributed to the finite size of the Fourier filter. Therefore, a horizontal slit filter may not necessarily produce a horizontal roll pattern as was also seen in Ref. [20]. In the present simulation, the horizontal 'roll' pattern is clearly tilted leading to the slight distortion of the square pattern. The nonideal character of the square pattern is also apparent in the corresponding experimental figure.

We were also able to show pattern manipulation using negative control. For manipulation of a hexagonal pattern, we used the same three-fold slit mask as in experiment (Fig. 4). Starting with a hexagonal pattern with two spots arranged on the vertical axis, we were able to turn the hexagon position by feeding back destructively unwanted components. As a re-
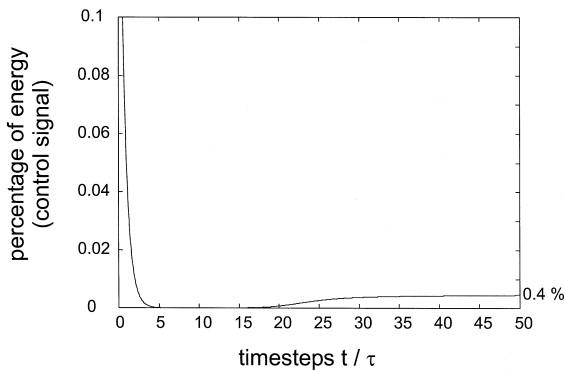
Fig. 5. Control signal fed back into the system as percentage of the energy with respect to the total energy in the pattern arm. Results of the numerical simulation leading to the right part of Fig. 4 .

sult, a rotated hexagonal pattern with two spots on the horizontal axis can be obtained. Here, as an important property of this control method, the control signal nearly vanishes when the desired state is selected. In Fig. 5, the control signal, i.e. the fraction of energy reinjected into the system relative to the total power in the pattern arm, is depicted. The signal declines from the initial to a very small value of 1.35×10^{-5} at $t/\tau \approx 9$ and then approaches an asymptotic value of about 0.4%.

4. Conclusion

We showed that manipulation and control of spontaneously formed patterns is possible by using positive and negative Fourier space control in an interferometric feedback system. This control scheme represents an improvement compared to previous methods and results since the pattern forming arm remains unchanged, while control is performed in an external arm that is fully separated from the first one. The power absorbed by the filter tends to a very small level when achieving control using the positive control method. Using this feedback scheme, we could excite rolls and squares in a parameter space region, where the only stable solution was a hexagonal one. Only a small control signal was applied to the system proving the effectiveness and minimally-invasive character of this method. We were also able to manipulate the position of a hexagonal pattern by negative control, and showed the minimization of the

corresponding control signal in this case. A numerical simulation was performed showing good agreement with experimental results despite the differences in the nonlinearity. Thus, we showed the efficiency of this method to control, manipulate and select spontaneously formed patterns. Only a small percentage of the energy of the system fed back as a manipulated control signal is necessary to change the output of the whole pattern forming system.

Acknowledgements

References

[1] D. Walgraef, Spatio-Temporal Pattern Formation, Springer, New York, 1995.
[2] M.C. Cross, P.C. Hohenberg, Rev. Mod. Phys. 65 (1993) 851.
[3] M.A. Vorontsov, W.B. Miller (Eds.), Self-Organization In Optical Systems and Applications in Information Technology, Springer, Berlin, 1995.
[4] G. D'Allesandro, W.J. Firth, Phys. Rev. Lett. 66 (1991) 2597.
[5] R. Macdonald, H.J. Eichler, Opt. Commun. 89 (1992) 289.
[6] B. Thüring, R. Neubecker, T. Tschudi, Opt. Commun. 102 (1993) 111.
[7] G. Giusfredi, J.F. Valley, R. Pon, G. Khitrova, H.M. Gibbs, J. Opt. Soc. Am. B 5 (1988) 1181.
[8] G. Grynberg, A. Maître, A. Petrossian, Phys. Rev. Lett. 72 (1994) 2379.
[9] T. Ackemann, W. Lange, Phys. Rev. A 50 (1994) R4468.
[10] J. Glückstad, M. Saffman, Opt. Lett. 20 (1995) 551.
[11] T. Honda, Opt. Lett. 18 (1993) 598.
[12] T. Honda, P.P. Banerjee, Opt. Lett. 21 (1996) 779.
[13] T. Honda, H. Matsumoto, M. Sedlatschek, C. Denz, T. Tschudi, Opt. Commun. 133 (1997) 293.
[14] C. Denz, M. Schwab, M. Sedlatschek, T. Tschudi, T. Honda, J. Opt. Soc. Am. B 15 (1998) 2057.
[15] M. Schwab, M. Sedlatschek, B. Thüring, C. Denz, T. Tschudi, Chaos, Solitons and Fractals 10 (1999) 701.
[16] P.M. Lushnikov, Zh. Éksp. Teor. Fiz. 113 (1998) 1122, [JETP 86 (1998) 614].
[17] J.V. Moloney, J. Opt. B: Quantum Semiclass. Opt. 1 (1999) 183.
[18] R. Martin, A.J. Scroggie, G.-L. Oppo, W.J. Firth, Phys. Rev. Lett. 77 (1996) 4007.

[19] G.K. Harkness, G.-L. Oppo, R. Martin, A.J. Scroggie, W.J. Firth, Phys. Rev. A 58 (1998) 2577.

[20] A.V. Mamaev, M. Saffman, Phys. Rev. Lett. 80 (1998) 3499.

[21] S. Juul Jensen, M. Schwab, C. Denz, Phys. Rev. Lett. 81 (1998) 1614.

[22] C. Denz, S. Juul Jensen, M. Schwab, T. Tschudi, J. Opt. B: Quantum Semiclass. Opt. 1 (1999) 114.

[23] G.K. Harkness, G.-L. Oppo, E. Benkler, M. Kreuzer, R. Neubecker, T. Tschudi, J. Opt. B: Quantum Semiclass. Opt. 1 (1999) 177.

[24] E. Ciaramella, M. Tamburrini, E. Santamato, Appl. Phys. Lett. 63 (1993) 1604.

[25] M. Saffman, A.A. Zozulya, D.Z. Anderson, J. Opt. Soc. Am. B 11 (1994) 1409.

[26] M. Schwab, C. Denz, M. Saffman, TOPS, vol. XXVII, Advances in Photorefractive Materials, Effects, and Devices, 1999, p. 424.

[27] B. Thüring, A. Schreiber, M. Kreuzer, T. Tschudi, Physica D 96 (1996) 282.

[28] M.A. Vorontsov, A.Yu. Karpov, Opt. Lett. 20 (1995) 2466.

[29] M.A. Vorontsov, A.Yu. Karpov, J. Opt. Soc. Am. B 14 (1997) 34.

[30] T. Ackemann, B. Giese, B. Schäpers, W. Lange, J. Opt. B: Quantum Semiclass. Opt. 1 (1999) 70.

[31] O. Sandfuchs, J. Leonardy, F. Kaiser, M.R. Belić, Opt. Lett. 22 (1997) 498.

[32] O. Sandfuchs, F. Kaiser, M.R. Belić, J. Opt. Soc. Am. B 15 (1998) 2070.