Selection of Unstable Patterns and Control of Optical Turbulence by Fourier Plane Filtering

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We report on selection and stabilization of transverse optical patterns in a feedback mirror experiment. Amplitude filtering in the Fourier plane is used to select otherwise unstable spatial patterns. Optical turbulence observed for nonlinearities far above the pattern formation threshold is stabilized by low-pass filtering of the optical power spectrum. Experimental observations obtained with a photorefractive nonlinearity are consistent with calculations based on a generic, cubic nonlinearity. [S0031-9007(98)05860-8]

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The spontaneous formation of periodic spatial patterns is a common feature of a range of physical systems that are excited beyond an instability threshold [1]. Many systems, including fluids, chemical reactions, granular media, and nonlinear optical devices, exhibit the following evolution. At low levels of excitation, where very little energy is fed into the system, a stable, spatially homogeneous, state exists. When the system is driven sufficiently hard, such that the threshold for amplification of either a discrete set, or continuous range, of spatial frequencies is exceeded, the spatially homogeneous state becomes unstable and structure begins to form. The symmetry properties of the emerging spatial structures depend both on bulk and boundary characteristics of the system. In two spatial dimensions hexagonal structures are generic. Considering a weakly excited system where the nonlinearity is described by a term that is third order in the amplitude of the pattern, hexagons are a natural consequence of phase matching in the plane. Analysis of the bifurcation structure of a mean field model of pattern formation reveals that other, nonhexagonal symmetries are valid solutions [2], yet they generally have smaller growth rates than the observed hexagonal structures.

Particularly from the point of view of the usefulness of pattern formation in the realm of optical information processing it is of interest both to extend the range of possible spatial structures, as well as to control which structures the system selects [3]. There are two broadly complementary approaches to the control of optical patterns. In the first, the geometrical structure of the system is modified such that other, nonhexagonal symmetries are preferred. In nonlinear optics the interaction of counterpropagating waves in a cubicly nonlinear medium with axial symmetry leads to generation of hexagonal patterns [4]. Modifying the basic interaction geometry in a variety of ways leads to a rich spectrum of nonhexagonal spatial structures [5].

A second approach is to stabilize the additional solutions that in principle exist in the original interaction geometry. Several recent works have considered techniques for stabilization and selection of optical patterns [6–8]. An important aspect of these methods is that, in the spirit of small perturbation techniques for controlling low-dimensional chaotic dynamics [9], they are essentially nonintrusive. When the targeted pattern is selected and stabilized the system geometry is unchanged, and the extra energy removed from, or added to, the system to achieve the stabilization tends to a small level.

The proposed techniques have been based on variants of spatially and temporally filtering the field in an optical feedback loop. A unique feature of optical pattern formation systems is that one has immediate access to the spatial power spectrum of the generated pattern, simply by looking in the back focal plane of a lens. This makes spectral control techniques particularly attractive in optical systems [10]. We demonstrate here, on the basis of experiments and numerical analysis, that a simple implementation of phase independent Fourier filtering techniques provides effective selection of unstable periodic patterns, as well as stabilization of optical turbulence that occurs far above the threshold for pattern formation. When the desired pattern is selected, or the optical turbulence is stabilized the energy removed by the filter tends to a small level, such that it becomes effectively nonintrusive.

The experimental geometry is shown in Fig. 1. A crystal of $KNbO_3$:Fe measuring $l = 5.2$ mm along the c axis was illuminated by a 30 mW beam at $\lambda = 532$ nm focused to a spot with a Gaussian diameter of about

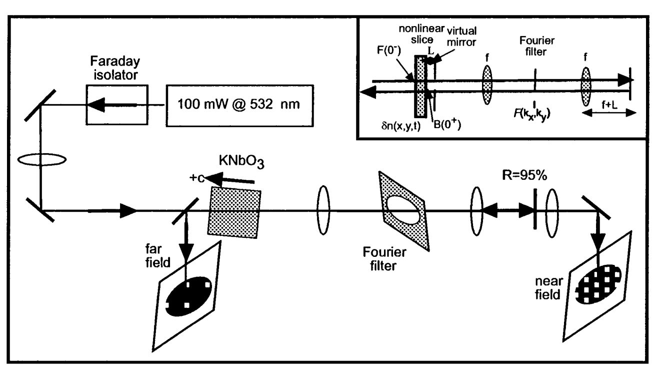

FIG. 1. Experimental geometry. The inset shows the geometry of the numerical model of Eqs. (1) and (2).

0.8 mm. The light was polarized parallel to the crystal a axis to take advantage of the r_{13} electro-optic coefficient. The counterpropagating pump beam was generated by a 95% reflecting mirror at the end of the cavity formed by two 150 mm focal length lenses. The Fourier transform of the field at the end of the crystal was located in between the two lenses (Fourier filter plane). The lenses were adjusted to give a virtual mirror position several millimeters past the end of the crystal. The instability in this geometry is due to the formation of reflection gratings with $k_g = 2k_0 n$ lying along the c axis, where $n = 2.3$ is the index of refraction of the crystal and $k_0 = 2\pi/\lambda$. In photorefractive media with a diffusion dominated charge transport mechanism the modulation of the material permittivity is shifted by $\pi/2$ with respect to the optical interference pattern. In this geometry an absolute instability is obtained in the presence of energy coupling between the counterpropagating waves [11], despite the lack of nonlinear phase shifts [12]. The crystal was oriented so that the reflected beam B was amplified by the pump F. Energy transfer measurements gave an estimate for the product of amplitude gain coefficient times medium length of $\gamma l \approx 6$, which is approximately twice the linear instability threshold in this system [12].

The left-most column of Fig. 2 shows experimental far-field images of pattern selection obtained with different amplitude filters inserted at the Fourier plane. With no filter a symmetric hexagonal pattern is obtained [Fig. 2(a)]. Restricting the filter to a thin slit leads to the formation of a stable roll pattern, with the wave vector parallel to the slit [Fig. 2(b)]. The roll pattern is a possible solution of a mean field model leading to hexagons [2], but is unstable in the full three-dimensional geometry. Additional observations have shown that blocking high frequency Fourier components leads to compression of the spectrum while blocking low frequency Fourier components leads to expansion of the spectrum. Changes to the spectrum can be understood qualitatively by recalling that the dispersion relation for counterpropagation instabilities has a minimum threshold leading to a well defined instability angle θ_0. Blocking the instability for $\theta \geq \theta_0$ leads to instability at angles $\theta < \theta_0$, while blocking the instability for $\theta \leq \theta_0$ leads to instability at angles $\theta > \theta_0$. The effect of anisotropic bandpass filtering is shown in Fig. 2(c). The filter prevents the formation of a symmetric hexagonal structure with the result that a stable rhombic, or squeezed hexagonal pattern that straddles the filter is formed. Finally, patterns with fourfold symmetry were selected using a four-quadrant mask as shown in Fig. 2(d).

A precise model of the reflection grating mediated interaction in diffusive photorefractive crystals can be formulated easily, but is expensive to solve numerically. Results published to date have been limited to a single transverse dimension [13]. We have therefore employed a generic model based on a transmission grating mediated interaction, in a thin slice of a sluggish medium with cubic nonlinearity [14]. The applicability of Fourier

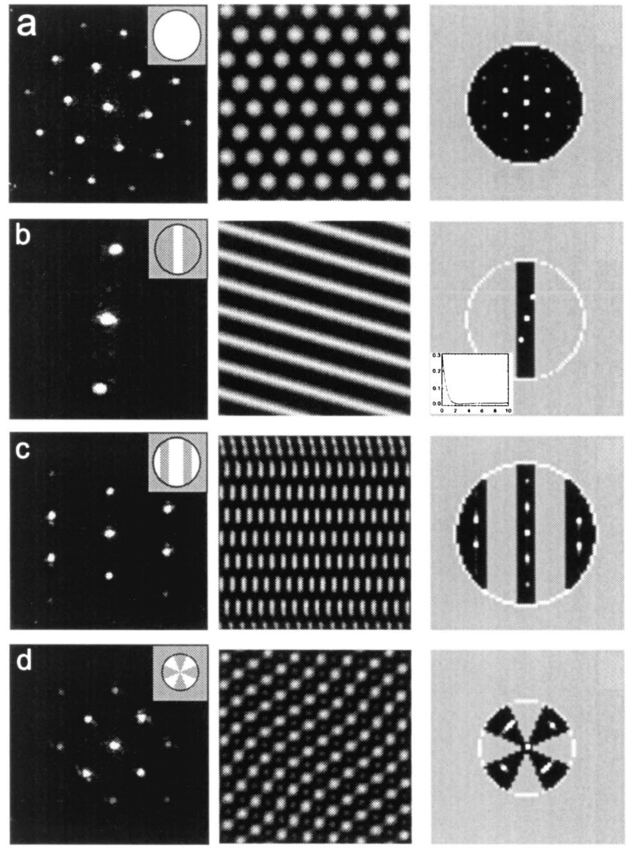

FIG. 2. Selection of unstable patterns by Fourier plane filtering: symmetric hexagons obtained with no filtering (a), rolls obtained with a linear filter (b), squeezed hexagons obtained with a bandpass filter (c), and squares obtained with a quadrant filter (d). The columns show from left to right exerimental far field, numerical near field, and numerical far field. The grey areas in the insets in the left-hand column depict regions where the spatial spectrum is blocked in the filter plane. Higher order components visible in frames (a), (c), and (d) actually lie outside the bandwidth of the filter, but are generated by readout of the photoinduced gratings by beam B as it passes back through the photorefractive medium. Numerical results were obtained with $\gamma/\gamma_{\text{th}} = 1.1$ in rows (a),(b), 1.5 in row (c), and 3.0 in row (d).

filtering techniques to pattern selection is not restricted to the photorefractive interaction used in the present experiments, and, as is seen below, the generic numerical model captures the essential features of the observed behavior.

In the simplified model the nonlinear refractive index δn is described by

$$\left(\tau \frac{\partial}{\partial t} + 1\right)\delta n = \mathcal{D}\nabla_\perp^2 \delta n + \gamma \frac{|F|^2 + |B|^2}{1 + |F|^2 + |B|^2}, \qquad (1)$$

where τ is the relaxation time of the medium, $\mathcal{D}$ is the diffusion coefficient, and F and B are the forward and backward propagating envelopes of the optical field, respectively. Calculations were performed with a small value of $\mathcal{D}$ in order to lift the degeneracy between higher order instability balloons [14]. The influence of the nonlinear medium located at $z = 0$ on the fields is given by $F(x,y,0^+) = F(x,y,0^-)e^{\imath\delta n}$ and $B(x,y,0^-) = B(x,y,0^+)e^{\imath\delta n}$. Free space diffraction, together with the effect of the Fourier filtering are calculated from

$$\frac{\partial F(x,y,z)}{\partial z} = \frac{i}{2k}\nabla_\perp^2 F(x,y,z)\,, \tag{2a}$$

$$\tilde{B}(k_x,k_y) = \tilde{F}(k_x,k_y)\mathcal{F}(k_x,k_y)\,, \tag{2b}$$

$$-\frac{\partial B(x,y,z)}{\partial z} = \frac{i}{2k}\nabla_\perp^2 B(x,y,z)\,, \tag{2c}$$

where tildes denote Fourier transformed functions and $\mathcal{F}(k_x,k_y)$ is a real function equal to zero, where the light is blocked by the Fourier mask, and 1 otherwise.

Steady-state solutions of Eqs. (1) and (2) with Fourier masks corresponding to those used experimentally are shown in Fig. 2. The near-field and far-field distributions obtained after an initial transient period are shown in the middle and right-hand columns, respectively. The numerical results demonstrate good qualitative agreement with the experimental observations. For rows (c) and (d) roll solutions were also observed close to the linear instability threshold. The displayed patterns were obtained by increasing the nonlinearity to 1.5 and 3 times threshold in rows (c) and (d), respectively. As discussed above the Fourier filter acts as a weak perturbation when the selected pattern is stabilized. This is shown in the small inset in Fig. 2(b) where the energy removed by the slit filter, when transforming from the hexagonal pattern of Fig. 2(a), to the roll pattern of Fig. 2(b), is shown to fall from an initial value of about 33% to 1.5% after several time constants. Additional calculations performed with a thick nonlinear medium also gave qualitatively similar results to those shown here.

When the nonlinear coefficient is much larger than the threshold value γ_{th}, optical turbulence is observed, as was pointed out in [15]. Experimental and numerical results are shown in Fig. 3 for $\gamma/\gamma_{\mathrm{th}} \sim 2$ in the experiment, and $\gamma/\gamma_{\mathrm{th}} \sim 3$ in the numerics. Looking at the far-field distributions in Fig. 3(a), where the normalized opening of the Fourier plane aperture is $r \equiv \theta_{\mathrm{filter}}/\theta_0 = 3$, it is evident that not only the first, but also the next higher order ring is strongly excited. In order to suppress the turbulence at high nonlinearities it is sufficient to reduce the width of the Fourier spectrum, much as higher order modes are eliminated in a laser by use of an intracavity aperture. Low-pass filtering to $r = 1.6$ is sufficient to stabilize the underlying hexagonal pattern, although a few defects remain, as shown in Fig. 3(c). Further filtering down to $r \approx 1$ results in a regular hexagonal pattern with essentially no visible defects. Additional experiments revealed that it was also possible to stabilize hexagons on the second ring by blocking the lowest order ring with high-pass filtering. Filtering the Fourier spectrum also affects the temporal dynamics in a similar fashion. The near-field intensity averaged over an area corresponding to one bright spot in a hexagonal pattern is highly irregular at large values of r, as shown in Fig. 4(a). Reducing r damps the fluctuations, so that only slow drifts associated with nonuniformities in the large scale structure remain. The numerical runs in Fig. 4(b) show a corresponding behavior.

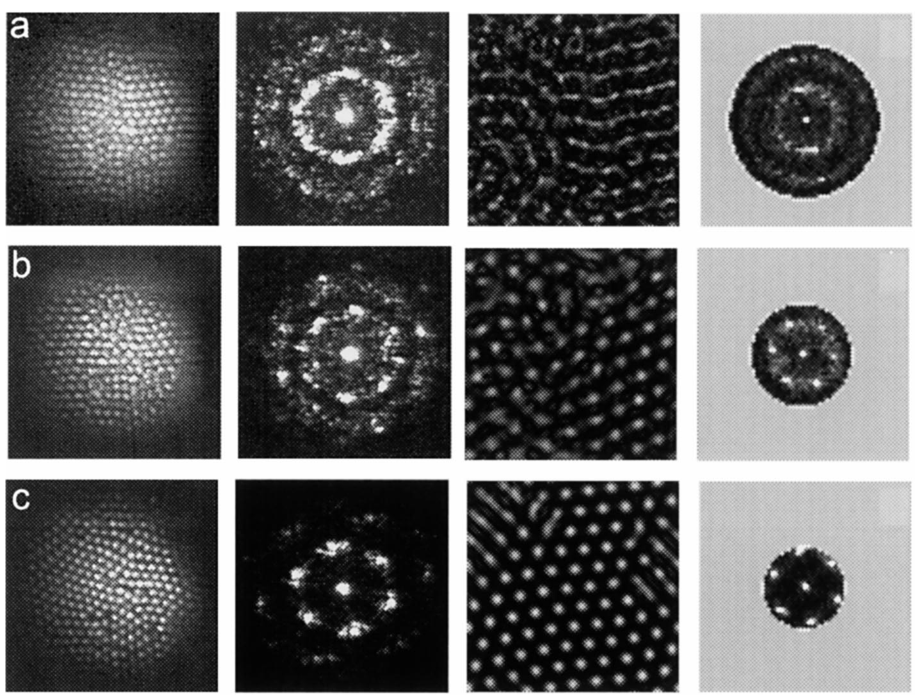

FIG. 3. Turbulence suppression by Fourier plane low-pass filtering. The normalized filter opening was $r = 3$, 2, and 1.6 in (a), (b), and (c), respectively. The columns show experimental near field, experimental far field, numerical near field, and numerical far field, from left to right.

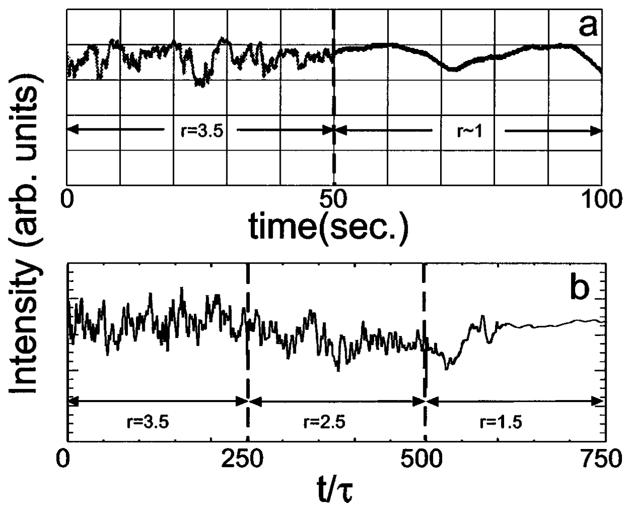

FIG. 4. Temporal variation of the near-field intensity obtained experimentally (a), and numerically (b).

In conclusion we have demonstrated the effectiveness of Fourier filtering for selection of transverse optical patterns, and suppression of turbulence far above threshold. The results shown here are rudimentary; we expect that future experiments based on more precise manipulation of the amplitude and/or phase distribution in the Fourier plane will allow sophisticated selection and stabilization techniques.

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