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Bound dipole solitary solutions in anisotropic nonlocal self-focusing media

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We find and analyze bound dipole solitary solutions in media with anisotropic nonlocal photorefractive material response. The dipole solutions consist of two elliptically shaped Gaussian-type beams separated by several diameters, and with a π phase shift between their fields. Spatial evolution of two initially round Gaussian beams and their convergence to the above dipole solution is demonstrated experimentally. [S1050-2947(97)51308-4]

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The possibility of creating two-transverse-dimensional [(2+1)D] soliton-type structures of light in nonlinear media is of considerable interest due to potential applications in optical information processing systems [1]. The dynamics of nonlinear propagation equations resulting in the formation of such structures can be very complex and may result in the generation of higher-order and multisoliton solutions. These solutions have been investigated extensively in the one-transverse-dimensional [(1+1)D] case [2]. A number of exact analytical methods have been developed, including the inverse scattering method, which provides an exact solution of an initial-value problem, Bäcklund transformations, and Lax pairs. Examples of higher-order bound soliton solutions include the formation of soliton-antisoliton pairs (kinks) [3], incoherently coupled bright and dark beams [4], and multisoliton solutions of a set of coupled high-order nonlinear Schrödinger equations [5].

In two-transverse dimensions the majority of nonlinear equations of interest are not integrable, and the corresponding initial or boundary-value problems are usually analyzed numerically. The analysis of a single-soliton solution if it does not exhibit a simplifying (e.g., radial) symmetry may require sophisticated tools [6]. The existence and properties of higher-order and multisoliton solutions in the (2+1)D case have therefore been investigated much less than in the (1+1)D case. Noteworthy results include calculation of nonlinear dipolar structures in the context of the Korteweg–de Vries equation [7], molecular matrices with impurities [8], and fluids [9]. Higher-order radially symmetric structures in media with cubic nonlinearity were given in [10], and their evolution in a saturating nonlinear medium studied in [11]. More complex localized solutions are described in [12].

In this paper we find and investigate (2+1)D dipole solitary solutions for a light beam in a self-focusing nonlocal anisotropic optical medium. This solution is a bound pair consisting of two elliptical beams spaced some distance apart that are in antiphase to each other (have a π relative phase shift). The above dipole solution is unique to the (2+1)D case and has no analogs in the (1+1)D case for neither anisotropic nor Kerr-type isotropic nonlinear media. Evidence of a related bound-pair solution for a defocusing nonlinearity was reported recently in [13]. We demonstrate experimentally the importance of the relative phase of the two lobes of the dipole in determining the subsequent spatial evolution. The particulars of our analysis pertain to a photorefractive nonlinear response, but general conclusions should be qualitatively applicable to other media.

Our analysis is based on the set of equations [14]

$$\left[\frac{\partial}{\partial x}-\frac{i}{2}\nabla^2\right]B(\vec{r})=i\,\frac{\partial\varphi}{\partial z}B(\vec{r}), \tag{1a}$$

$$\nabla^2\varphi+\vec{\nabla}\ln(1+|B|^2)\cdot\vec{\nabla}\varphi=\frac{\partial}{\partial z}\ln(1+|B|^2) \tag{1b}$$

that describe time-independent propagation along the coordinate x in the presence of an electric field applied along z. Here $B(\vec{r})$ is the amplitude of the optical beam, $\vec{\nabla}=\hat{y}(\partial/\partial y)+\hat{z}(\partial/\partial z)$ is the transverse gradient, and φ is the electrostatic potential induced by the beam with the boundary conditions $\vec{\nabla}\varphi(\vec{r}\rightarrow\infty)\rightarrow\vec{0}$. Equations (1) are written in dimensionless coordinates. The electromagnetic intensity is normalized to the characteristic saturation intensity so that the saturation intensity in Eq. (1b) is equal to unity. More details about the derivation of Eqs. (1) and the normalizations used can be found in Refs. [14,15]. Theoretical analysis and experimental observations of single-soliton solutions of Eqs. (1) were discussed in Ref. [16].

Equations (1) are highly anisotropic [14,17] and do not allow radially symmetric solitary solutions, thereby necessitating explicit treatment of both transverse coordinates. Their second hallmark is the nonlocality of Eq. (1b) for the nonlinear change of the refractive index. An immediate consequence of this fact is that both single and multisoliton (2+1)D solitary solutions of Eqs. (1) have exponential asymptotics for the electromagnetic field, but algebraic asymptotics for the nonlinear refractive index of the medium, and in this sense can be called semialgebraic. Note that typical soliton solutions of nonlinear propagation equations have exponentially decaying asymptotics at infinity. One notable

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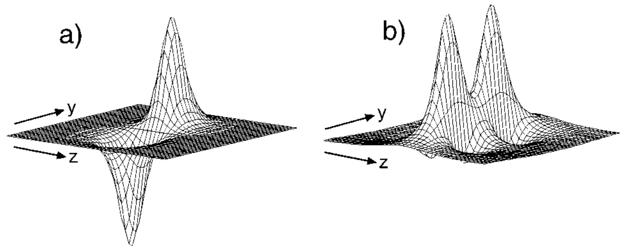

FIG. 1. Spatial distribution of (a) the field and (b) the nonlinear refractive index for the dipole solitary solution of Eqs. (1).

exception is the solutions of the Benjamin-Ohno equation that are algebraic in nature (see, e.g., Ref. [18]).

Solitary solutions of Eqs. (1) correspond to the ansatz $B(\vec{r})=b(y,z)\exp(i\lambda x)$, where λ is a real propagation constant. The resulting equations constitute an eigenvalue-eigenfunction problem with the eigenvalues λ forming a continuous set. The eigenfunctions $b(y,z)$ have been found, in the same way as in Ref. [16], using the iterative procedure of Petviashvili [19] (see also [7]). Note that this approach is exact and does not imply any approximations. To find dipole solitary solutions of Eqs. (1), starter functions for the iteration procedure have been specified as a pair of Gaussian beams with a π mutual phase shift spaced some distance apart.

Figure 1 illustrates (a) the distribution of the electromagnetic fields and (b) the nonlinear refractive index $\nu=\partial\varphi/\partial z$ corresponding to the dipole solitary wave solution of Eqs. (1) when the maximum intensity of the solution $I_m=1$, where $I_m\equiv|b|^2$ evaluated at the location of the maximum intensity of one of the dipole peaks. The solution is a pair of coupled, elliptically shaped (2+1)D beams that are spaced some distance apart and aligned along the y axis. The fields of each of the components comprising the pair have the same magnitude but different signs.

In Fig. 2 we present the diameters d_y and d_z of each of the beams comprising the dipole solution and the separation L_{sep} between them as functions of the maximum intensity I_m. The diameters have been calculated at the 1/2 level of the maximum intensity of the solution. Both of the intensity peaks comprising the dipole solution turn out to be smooth Gaussian-type beams that are narrower along the coordinate z and wider in the perpendicular direction. This fact reflects the dominant role of the anisotropy in the formation of the dipole solution. In the weak saturation limit $I_m \ll 1$, both the values of the diameters and the separation increase as the maximum intensity decreases. In the opposite limit of large saturation $I_m \gg 1$, they are logarithmically proportional to I_m. A shallow minimum is reached between these two limiting cases. The propagation constant λ was found to increase monotonically with the power of the dipole solution, which is indicative of its stability [20].

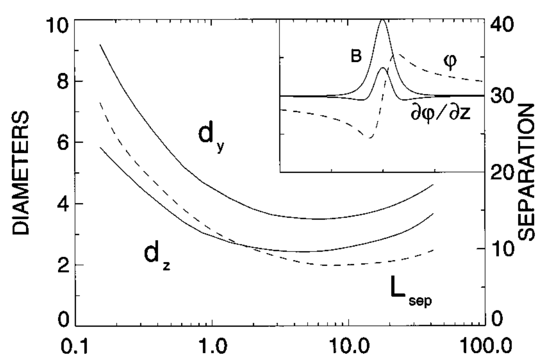

FIG. 2. Diameters and the separation between the beams comprising the dipole solitary solution versus its maximum intensity. The inset shows cross sections of the field, the refractive index, and the potential along the z axis.

The inset shows cross sections of the electromagnetic field (B), the refractive index ($\partial\varphi/\partial z$), and the potential φ taken along the z axis through the maximum of one of the intensity peaks for the parameters of Fig. 1. Note the slow power-law decay of the potential as z tends to infinity. The asymptotic structure of the potential at large distances is of the form $\varphi \propto z/\sqrt{y^2+z^2}$. This structure implies that the refractive index $\nu=\partial\varphi/\partial z$ has both positive (focusing) and negative (defocusing) regions; the latter are situated on both sides of the beams along the plus-minus directions of the z axis.

Numerical analysis of spatial dynamics and convergence properties of the dipole solution has been carried out for a pair of Gaussian beams spaced some distance apart along the y axis. The analysis indicates that the dipole solitary solution is an attractor of the set of equations (1). Initial distributions of the field in the vicinity of the dipole solitary solution converge to this solution. Note that this is not the only attractor of the system, since Eqs. (1) also contain the lowest-order single-beam soliton solution [16]. A unique manifestation of the formation of a bound pair by two antiphased Gaussian beams is the dynamics of their separation distance. Numerical runs show that if two beams are launched too close to one another, they repel each other; if initially they are too far away, they attract each other. Asymptotically the pair converges to the dipole solution via damped spatial oscillations of both the diameters and separation between the beams around the asymptotic equilibrium state. The rate of convergence depends primarily on the value of the normalized characteristic intensity I_m of the beams, in the same way as for convergence to the lowest-order soliton solution [16]. Both the low- and high-saturation limits $I_m \ll 1$ or $I_m \gg 1$ are unfavorable for observing photorefractive solitons since the convergence rate is slow (see Ref. [16]). The best regime corresponds to $I_m \simeq 1$ when the length of the spatial transient is minimal.

The convergence of two initially round Gaussian beams to the dipole solitary solution in this moderate saturation regime is illustrated by Fig. 3, which shows spatial evolution of the beam diameters and their separation distance for $I_m = 1.5$. The initial separation distance is larger than the equilibrium soliton separation, and the beams attract each other, as seen from the separation distance changing from 12 at the input to 10.6 at the output. For initial separation distances smaller than that of the soliton solution the beams repel each other. The value of the normalized propagation distance for the experiment described below is about 30. The rate of attraction and/or repulsion is relatively small, so in experiment we have to adjust the initial separation close to that of a soliton solution.

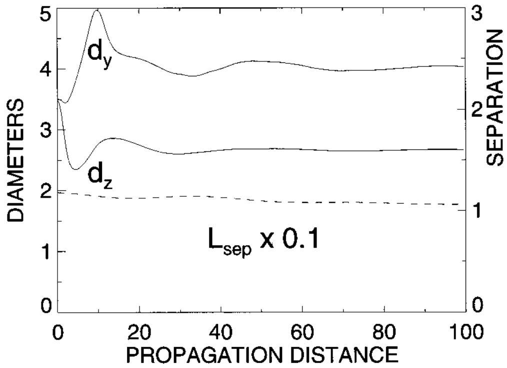

FIG. 3. Evolution of the diameters and separation of input antiphase Gaussian beams with propagation distance in the moderate saturation regime.

If the relative phase between the beams is close, but not equal to 180°, the spatial evolution of the beams is accompanied by oscillating exchange of their intensities. For large deviations from the antiphased conditions the evolution of the pair changes quite dramatically and the dipole solution cannot be reached. In particular, a pair of Gaussian beams with zero relative phase coalesces.

Excitation of a dipole soliton was demonstrated using an experimental arrangement similar to that used in Ref. [15]. As shown in Fig. 4, a 10-mW He-Ne laser beam ($\lambda = 0.63$ μm) was passed through a system of lenses controlling the size of the beam waist, and a beam splitter arrangement was used to create two parallel circular beams with variable separation and adjustable relative phase. The beams were directed into a photorefractive crystal of SBN:60 doped with 0.002 wt. % Ce. The beam propagated perpendicular to the crystal $\hat{c}$ axis, and was polarized along it to take advantage of the largest component of the electro-optic tensor of SBN r_{33}. The experimentally measured value of the latter was found to be $r_{33} \approx 210$ pV/m. The crystal measured 20 mm along the direction of propagation, and was 5 mm wide along the $\hat{c}$ axis. A 1-kV dc voltage was applied along the

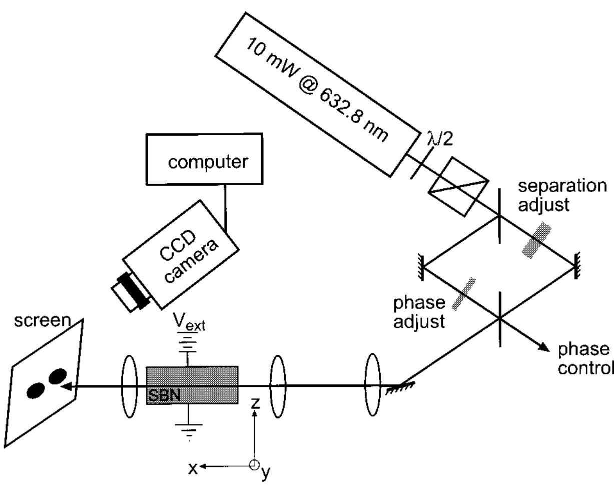

FIG. 4. Experimental setup. The half-wave plate and beam splitter were used for controlling the laser intensity. The glass plates in the arms of the interferometer were rotated to adjust the separation and phase of the beams. The relative phase was controlled by observing the unused interferometer output.

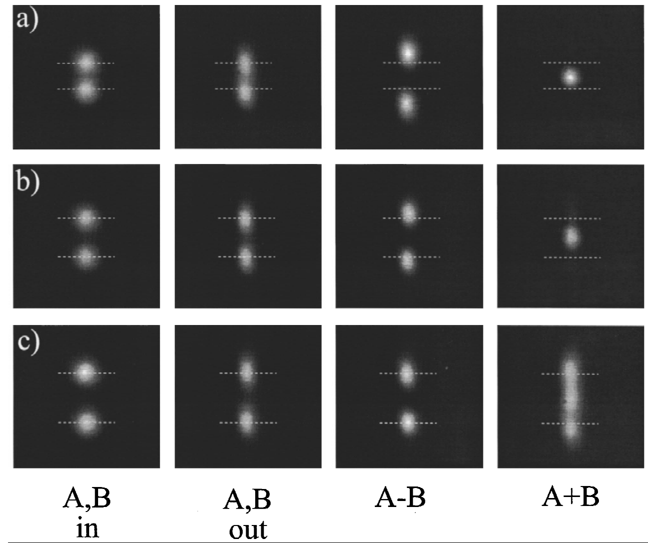

FIG. 5. Experimental observation of excitation of a dipole (see main text for explanation). Each frame depicts a region of size 250 by 250 μm, and the z coordinate is horizontal.

$\hat{c}$ axis and the effective saturation intensity was varied by illuminating the crystal from above with incoherent white light. The experiments were carried out in a moderate saturation regime with $I_m \simeq 0.5$, and the power in each beam was 40 μW. Images of the beam at the input and output faces of the crystal were recorded with a charge-coupled-device camera.

A signature of the formation of a bound dipole is the sensitive dependence on the relative phase of the input beams. Figure 5 shows the the input and output intensity distributions for input separations of (a) 44 μm, (b)65 μm, and (c) 83 μm. The leftmost column labeled ''A,B in'' shows images of the intensity at the input surface of the crystal. The pictures were made by recording the intensity of each beam separately, and then adding the digitized pictures with a computer. The next column ''A,B out'' shows output distributions, created in the same way, that were obtained by allowing beams A and B to propagate one at a time through the crystal. Comparison of the first two columns demonstrates that the input beams were parallel (equal separation at input and output faces) and that they converged to elliptically shaped solitary solutions [16]. The input beams had a diameter of 33 μm full width at half maximum. The elliptically shaped output beams had measured diameters of $d_z = 20$ μm and $d_y = 30$ μm.

Column ''A-B'' shows the output recorded when both beams were present with a relative input phase of π. When the input separation is too small the beams repel each other, and when the separation is too large the beams attract. The amount of repulsion can be large, as seen in (a) and (b), since the interaction between closely spaced beams is large. On the other hand, the amount of attraction is relatively small, since it occurs for widely spaced beams for which the interaction is weak. A small amount of attraction is visible in row (c), but the effect is weak, and is close to the accuracy of our measurements. The attraction was observed clearly by blocking one of the input beams, which resulted in the other beam moving slightly further away from the blocked beam. For row (c) we measure $L_{sep}/d_z = 83/20 = 4.2$, which is close to the ratio $L_{sep}/d_z \simeq 4.6$ given in Fig. 3 for $I_m = 0.5$. The measured ellipticity of each of the output peaks was 1.5, in agreement with Fig. 3. We conclude, therefore, that row (c) demonstrates excitation of a dipole pair consisting of two elliptically shaped beams with π relative phase shift.

Finally, for in-phase input beams (column ''A+B'') we observe fusion for small input separation [(a) and (b)], and formation of an elongated beam for large input separation (c). Based on numerical simulations we expect the beams in (c) to eventually fuse for a sufficiently long nonlinear medium.

In conclusion, we have found bound dipole solitary solutions to the problem of anisotropic nonlocal self-focusing in bulk photorefractive media. Properties of these solutions have been analyzed in detail. No rigorous asymptotic stability analysis of the above solutions has been performed. Nevertheless numerical runs starting from a pair of Gaussian beams with different input conditions and/or exact dipole solitary solution demonstrate convergence to these solutions and their apparent stability. Experimental results for the spatial evolution of Gaussian beams confirm the numerical simulations indicating convergence to and the existence of a higher-order solitary wave consisting of an aligned bound dipole pair.

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