

Modulational instability and pattern formation in the field of noncollinear pump beams

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We study pattern formation that is due to modulational instability of noncollinear counterpropagating beams. Angular misalignment of the pumps over a broad range leads to the generation of lines oriented perpendicular to the plane of the pump beams. A theoretical expression for the angular separation of the lines is in close agreement with observations. Outside this range, for both small and large misalignments, we observe squeezed hexagonal patterns. © 1997 Optical Society of America

There are several reasons for the current high level of interest in pattern formation in optical systems. Nonlinear optics has proved to be an attractive medium for studying symmetry-breaking and dynamics in nonequilibrium systems. Results obtained in optics parallel corresponding observations from fluid and other systems, as in the case of formation of hexagons,¹ but also reveal exotic spatiotemporal structures not seen elsewhere.²⁻⁶ There are also intriguing prospects for applications of pattern formation to information processing.⁷⁻⁹

A generic mechanism leading to optical pattern formation is the transverse modulational instability of pump beams counterpropagating in a cubic nonlinear medium. For collinear pumps there is cylindrical symmetry about the z (propagation) axis, and hexagonal patterns are typically observed.^{1,10-12} It was shown previously that misalignment of the pump beams breaks the phase-matching condition in the plane that contains the pumps and leads to a collapse of the hexagonal pattern into a one-dimensional roll pattern in the plane perpendicular to the plane of the pump beams.¹ If the instability is restricted to the plane of misalignment, e.g., by use of elliptically shaped pump beams, a far-field frequency shift and a corresponding near-field drift motion that are proportional to the misalignment angle are obtained.^{13,14}

We show here that introducing pump misalignment, while permitting instability in both transverse dimensions, leads to the generation of previously unobserved patterns. For intermediate values of the misalignment we observe lines perpendicular to the misalignment plane. Modulational instability of the lines leads to a two-dimensional transverse pattern with rhombic symmetry. For small misalignment we observe both rolls and squeezed hexagons, and for relatively large misalignment, in which the angle at which the lines are generated is comparable with the normal angle for hexagon generation, we observe squeezed hexagons.

The generation of lines can be explained in the context of the simplified model of a thin nonlinear slice with a feedback mirror.¹⁵ Consider counterpropagating pumps given by $F = F_0(z)\exp(iqx)\exp \times [-i(q^2/2k_0)z]\exp(ik_0z)$ and $B = B_0(z)\exp(iqx)\exp \times$

$[i(q^2/2k_0)z]\exp(-ik_0z)$, corresponding to a tilt of the input beam by angle $\phi = q/k_0$, where $k_0 = 2\pi/\lambda$ is the vacuum wave number. We consider the instability of F and B with respect to angularly tilted sidebands of the form $f_{\pm}(z)\exp(\pm ik_{\pm}x)$ and $b_{\pm}(z)\exp(\pm ik_{\pm}x)$. Specification of the boundary conditions introduces diffraction into the linear stability analysis in the form

$$b_{\pm}(0) = f_{\pm}(0)\exp(i2lk_{\pm}), \quad (1)$$

where $k_{\pm} = (k_{\perp} \pm q)^2/2k_0$ and l is the distance from the nonlinear slice to the mirror. For no misalignment ($q = 0$) the normal roll instability appears at $\theta_{0,\text{slice}} = k_{\perp}/k_0 = \sqrt{\lambda/(4l)}$ in a self-focusing medium.¹⁵ In the presence of finite misalignment the + and - sidebands experience equal diffractive phase shifts modulo 2π , provided that $2l(k_+ - k_-) = 2m\pi$, where m is an integer. This implies that, provided that the threshold for the formation of rolls in the aligned system is reached, there will be the possibility of generation of lines at angles

$$\theta_m = m \frac{\pi}{2k_0 l \phi}. \quad (2)$$

Note that the angular separation of the lines is inversely proportional to the misalignment of the input beam.

Formation of lines in the field of noncollinear pumps is a general mechanism that is not restricted to the thin-slice model. The arguments leading to Eq. (2) for the angular separation of the lines are valid for the thin-slice geometry. For a thick nonlinear medium with a feedback mirror (as in our experiments) or for an externally pumped thick medium, coupled equations for the sidebands of misaligned pumps should be solved. Based on the known similarities between pattern formation in thick media and in the thin-slice model, we do not expect the presence of a thick medium to change the situation significantly. The validity of our model in describing an experiment with a thick nonlinear medium is demonstrated by the good agreement between Eq. (2) and experiment, demonstrated in Fig. 2 below.

We have observed this instability by using the geometry shown in Fig. 1. A crystal of $\text{KNbO}_3:\text{Fe}$

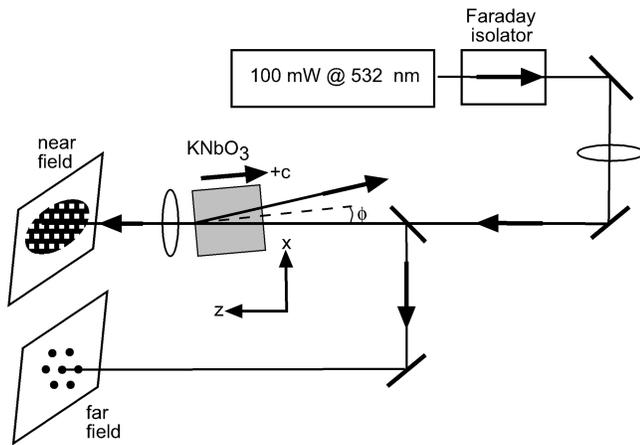


Fig. 1. Experimental setup. Similar near- and far-field pictures were observed from either end of the crystal.

measuring 5.2 mm along the c axis was illuminated by a 30-mW beam at 532 nm focused to a spot of Gaussian diameter 0.8 mm. We polarized the light parallel to the a axis to take advantage of the r_{13} electro-optic coefficient. The counterpropagating pump beam was generated by the 16% reflection from the uncoated back surface of the crystal. The instability in this geometry is due to the formation of reflection gratings with $k_g = 2k_0n$ lying along the c axis, where $n = 2.3$ is the index of refraction of the crystal. The crystal was oriented such that the reflected beam B was amplified by pump beam F . The estimated intensity gain coefficient for the crystal that we used was $\Gamma l \approx 12$.

Figure 2 shows the far-field patterns and the measured dependence of the angle θ_1 on the input misalignment ϕ . Despite the simplicity of the theoretical model there is good agreement with the experimentally observed angular separation of the lines. The threshold for modulational instability of the lines at $\pm\theta_1$ is higher than for the central line because a smaller fraction of the total light intensity participates in the generation of the extra lines compared with the central line.

The vertical lines have a clearly visible ring structure. The rings are due to the presence of Fabry–Perot resonances in the crystal, which was prepared with the c faces parallel to ~ 0.2 mrad. Small changes in pump-beam intensity, and hence in the amount of absorption-driven heating in the crystal, change the crystal's length and the angular position of the Fabry–Perot resonances of the cavity.⁶ When a Fabry–Perot resonance coincides with $\theta_0 \approx \sqrt{n\lambda/l}$, which is the angle for roll instability in a thick medium with nonlinear energy coupling,^{16,17} the effective threshold is reduced¹⁸ and the extra lines also become modulationally unstable, as Fig. 3 shows. The roll instability has angular scale θ_0 , whereas the lines are angularly separated by θ_1 , so the resulting two-dimensional pattern is not rotationally symmetric. The relative phase of the instability along the three lines is not fixed, so two-dimensional tilings with rhombic symmetry, as seen in Fig. 3(a), but also rectangular symmetry can be observed. The near-field pattern shown in Fig. 3(b) moved along x , as was observed previously.¹³

At small values of ϕ the angle between the lines is so large that the overlap with the pump beams over the length of the crystal is reduced, so the excitation of the lines becomes weaker. Under these conditions the possibility of excitation of the lines also depends on the cavity tuning, as shown in Fig. 4, obtained for fixed $\phi = 0.83$ mrad but variable crystal temperature. Changes both in the degree of excitation of the lines and in the symmetry of the central pattern are seen. Note that the hexagons are squeezed by $\sim 5\%$ along the x axis.

At large values of ϕ where θ_1 becomes comparable with θ_0 the lines disappear, and instead we obtain squeezed hexagonal structures, as shown in Fig. 5. These hexagons were squeezed by $\sim 10\%$, and the squeezing took place along either the x or the y axes, depending on fine adjustment of ϕ . The reappearance of hexagons, despite large pump-beam misalignment, can be interpreted as a cooperative instability arising from the similarity of θ_0 and θ_1 . Previous experiments with hexagon formation in KNbO_3 have shown that the hexagonal pattern can rotate about the axis. It has been shown by Honda¹⁹ that the rotation is due to, and can be controlled by, gradients in the

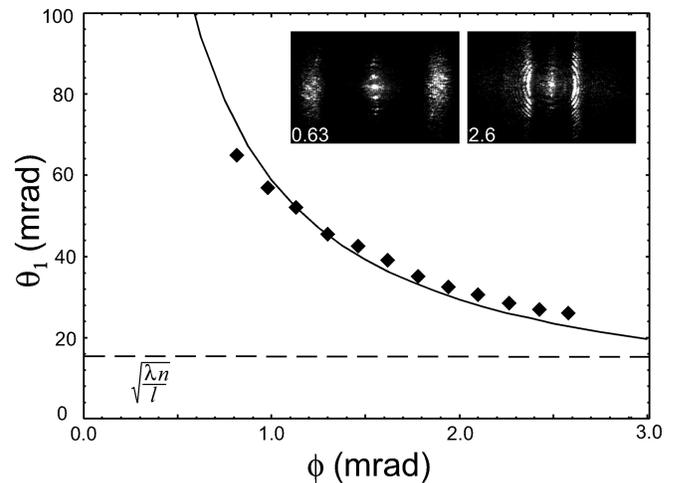


Fig. 2. Measured dependence of the instability angle θ_1 on the misalignment angle ϕ . The solid curve is Eq. (2) [assuming that the condition $2l(k_+ - k_-) = 2m\pi$ holds at the input end of the crystal, Eq. (2) becomes $\theta_m = m\lambda n/(4l\phi)$, where θ_m and ϕ are external angles], and the dashed line shows the angle of the normal roll instability. The insets show the far-field patterns observed for $\phi = 0.63$ and $\phi = 2.6$ mrad.

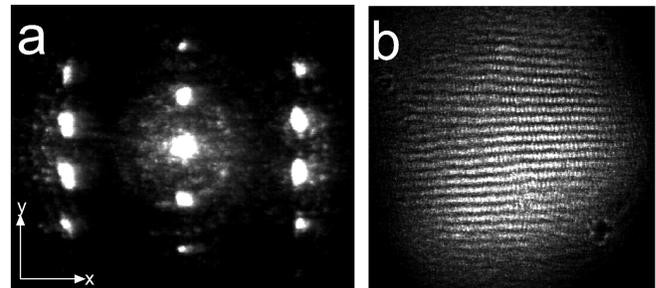


Fig. 3. (a) Far-field and (b) near-field rhombic patterns observed for $\phi = 1.9$ mrad.

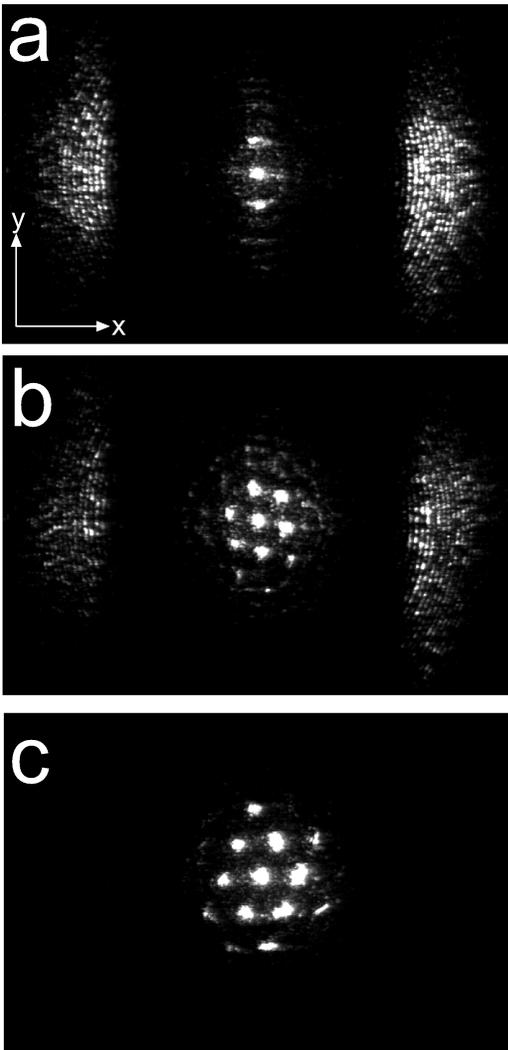


Fig. 4. Variation of the far-field pattern for fixed $\phi = 0.83$ mrad and different resonator tunings.

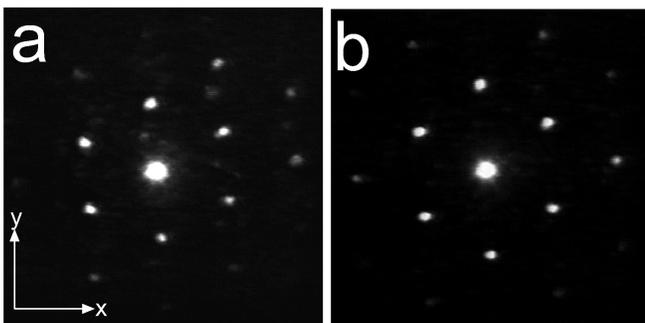


Fig. 5. Hexagons squeezed along (a) y and (b) x observed for large values of ϕ .

pump-beam intensity. The squeezed hexagons have a preferred angular orientation that inhibits rotation.

We observed instead a periodic rocking motion consisting of initial formation of a pattern with the alignment seen in Fig. 5, rotation by ~ 15 deg, followed by a rapid shift back to the initial orientation, after which the sequence was repeated.

In summary, we have observed some new optical patterns obtained with misaligned pump beams. We note finally that very similar results are obtained when the misaligned second pump beam is generated by a tilted high-reflectivity mirror placed directly behind the crystal.

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