Transfer of temporal fluctuations in photorefractive two-beam coupling

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Transfer of temporal fluctuations between the signal and pump beams in diffusion dominated photorefractive two-beam coupling is studied experimentally. The dependence on the gain, beam intensity ratio, and frequency of the fluctuations is found to agree well with a linearized analysis. The transfer of perturbations is frequency dependent at low frequencies, and becomes constant at frequencies large compared to the inverse material time constant. We discuss the possibility of pump noise suppression when amplifying weak signals.
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The temporal dynamics of beam coupling in photorefractive media have been the subject of numerous studies.[1–8] Since no general analytical solution is available, either numerics or approximate analysis must be used. In the undepleted pump limit, a transfer function is sufficient to describe the temporal behavior of the signal,[7] while for arbitrary pump to signal ratios a transfer matrix describing the coupling of fluctuations between the pump and signal beams is available.[9,10] An analytical study of the stability of several photorefractive resonator circuits based on the transfer matrix formalism was found to be consistent with experimental observations.[10] Nonetheless, a direct experimental measurement of the transfer matrix elements has been lacking. In the present letter, we demonstrate agreement between measurements of the four matrix elements and the transfer matrix formalism. The behavior of the transfer matrix coefficients for frequencies small compared to the inverse of the dielectric relaxation time of the material leads to some potentially useful aspects of frequency dependent two-beam coupling. In particular, we show that it is possible to amplify a weak signal with a noisy pump beam, such that the signal to noise ratio of the amplified signal is higher than that of the initial pump beam.

Two-beam coupling in photorefractive media is described by Eqs. (1a) and (1b)

$$\frac{\partial s}{\partial z}=gp, \quad \frac{\partial p}{\partial z}=-g^*s, \tag{1a}$$

$$\left(\tau\frac{\partial}{\partial t}+1\right)g=\frac{\Gamma}{2}\frac{sp^*}{I}, \tag{1b}$$

where s and p are the signal and pump amplitudes, g is the grating amplitude, and the time constant τ scales inversely with the total intensity $I=|s|^2+|p|^2$. To solve Eqs. (1a) and (1b), we write the signal and pump beams as the sum of constant and fluctuating parts: $s(z,t)=s^{(0)}(z)+Re[\delta s(z)e^{i\Omega t}]$, and $p(z,t)=p^{(0)}(z)+Re[\delta p(z)e^{i\Omega t}]$, where Ω is the fluctuation frequency. The solution of the dc amplitudes can be written in the compact form[11]

$$\begin{bmatrix} s^{(0)}(z) \\ p^{(0)}(z) \end{bmatrix}=\begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix}\begin{bmatrix} s^{(0)}(0) \\ p^{(0)}(0) \end{bmatrix}, \tag{2}$$

where $\Theta=\arctan(e^{\Gamma z/2}/\sqrt{m})-\arctan(1/\sqrt{m})$, and $m=|p^{(0)}(0)/s^{(0)}(0)|^2$.

Linearizing Eqs. (1a) and (1b) with respect to the perturbations and solving gives for the fluctuations

$$\begin{bmatrix} \delta s(z) \\ \delta p(z) \end{bmatrix}=\mathbf{T}(z)\begin{bmatrix} \delta s(0) \\ \delta p(0) \end{bmatrix}, \tag{3}$$

where the transfer matrix $\mathbf{T}$ relates the fluctuations before and after the beam interaction in the crystal. The matrix elements are given by[9,10]

$$T_{11}=\cos(\Theta)\frac{1+me^{-\Gamma z/2}e^H}{1+me^{-\Gamma z/2}}, \tag{4a}$$

$$T_{12}=\sin(\Theta)\frac{1-e^{-\Gamma z/2}e^H}{1-e^{-\Gamma z/2}}, \tag{4b}$$

$$T_{21}=-\sin(\Theta)\frac{e^{-\Gamma z/2}-e^H}{e^{-\Gamma z/2}-1}, \tag{4c}$$

$$T_{22}=\cos(\Theta)\frac{e^H+me^{-\Gamma z/2}}{1+me^{-\Gamma z/2}}, \tag{4d}$$

with $H=\{\ln[(1+m)/(1+me^{-\Gamma z})]-\Gamma z/2\}/(1+\Omega\tau)$. Note that for $\Omega\tau\to\infty$ the factor H vanishes and the $\mathbf{T}$ matrix reduces to the steady state solution Eq. (2). This means that high frequency fluctuations scatter off of the grating formed by the low frequency structure of the fields, as they would from a passive, frequency independent, beamsplitter. It is in this high frequency regime that photorefractive crystals have been used as adaptive beam combiners for homodyne and heterodyne detectors.[12–15]

Direct detection of a signal beam with weak modulation at frequency Ω leads to a photocurrent with ratio between the fluctuating and dc components given by $R_s(0)=\delta s(0)/s^{(0)}(0)$. The output fluctuation ratios are given by

$$R_s(z)=\frac{T_{11}}{\cos(\Theta)+\sqrt{m}\,\sin(\Theta)}R_s(0)+\frac{T_{12}}{\cos(\Theta)/\sqrt{m}+\sin(\Theta)}R_p(0), \tag{5a}$$

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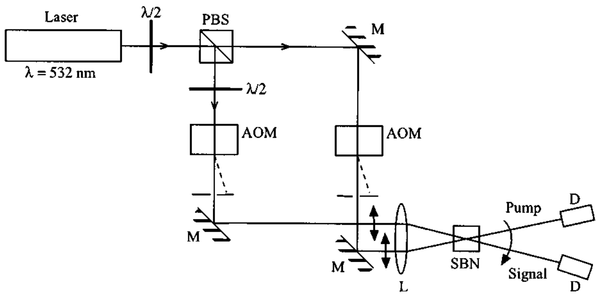

FIG. 1. Experimental setup.

$$R_p(z)=\frac{T_{21}}{\sqrt{m}\,\cos(\Theta)-\sin(\Theta)}\,R_s(0)+\frac{T_{22}}{\cos(\Theta)-\sin(\Theta)/\sqrt{m}}\,R_p(0). \tag{5b}$$

The individual matrix elements T_{ij} can then be isolated by setting $R_s(0)$ or $R_p(0)$ separately equal to zero.

The matrix elements were measured with the two-beam coupling setup shown in Fig. 1. Two linearly polarized beams from a frequency doubled Nd:YAG laser at 532 nm were passed through acousto-optic modulators (AOM) that diffracted a small part of the beams at a predetermined frequency. The directly transmitted beams consisted of a dc component together with a small sinusoidally varying amplitude modulation. The beams were then coupled in a SBN ($Sr_2BaNb_2O_6$) crystal. The grating vector was parallel to the crystal c axis and the crystal measured 5 mm along the direction of propagation. The total intensity transmission factor was 0.6 which included Fresnel and absorptive losses. The beams were focused into the crystal with a 100 mm lens giving an external full crossing angle of ~8° and spots of diameter 65 μm full width at half-maximum (FWHM). The tightly focused beams gave a relatively short interaction region, and hence a modest coupling $\Gamma l \sim 2-3$. The advantage of focusing the beams is the increase of intensity in the interaction volume, leading to a shorter time constant, and a faster frequency response. Measurements were made with total power incident on the crystal of 6.5 and 55 mW leading to time constants τ of about 0.25 and 0.05 s, respectively. The time constant was measured by frequency shifting one of the beams with a piezomirror, and measuring the variation of coupling strength with frequency shift. The beams were then detected with silicon photodiodes and the dc and fluctuating amplitudes measured with a spectrum analyzer. In order to distinguish between the different matrix elements of Eqs. (4a)–(4d), only one of the input beams was modulated at a time. The modulation frequency was tuned between 0 and 20 inverse time constants. The coupling coefficient was determined by measuring the transmitted signal beam with and without the pump present, and using Eq. (2).

The measured and the calculated matrix elements are compared in Fig. 2 for the low intensity experiment ($P=6.5$ mW), and input beam ratios of $m=0.1$, 1.0, and 10.

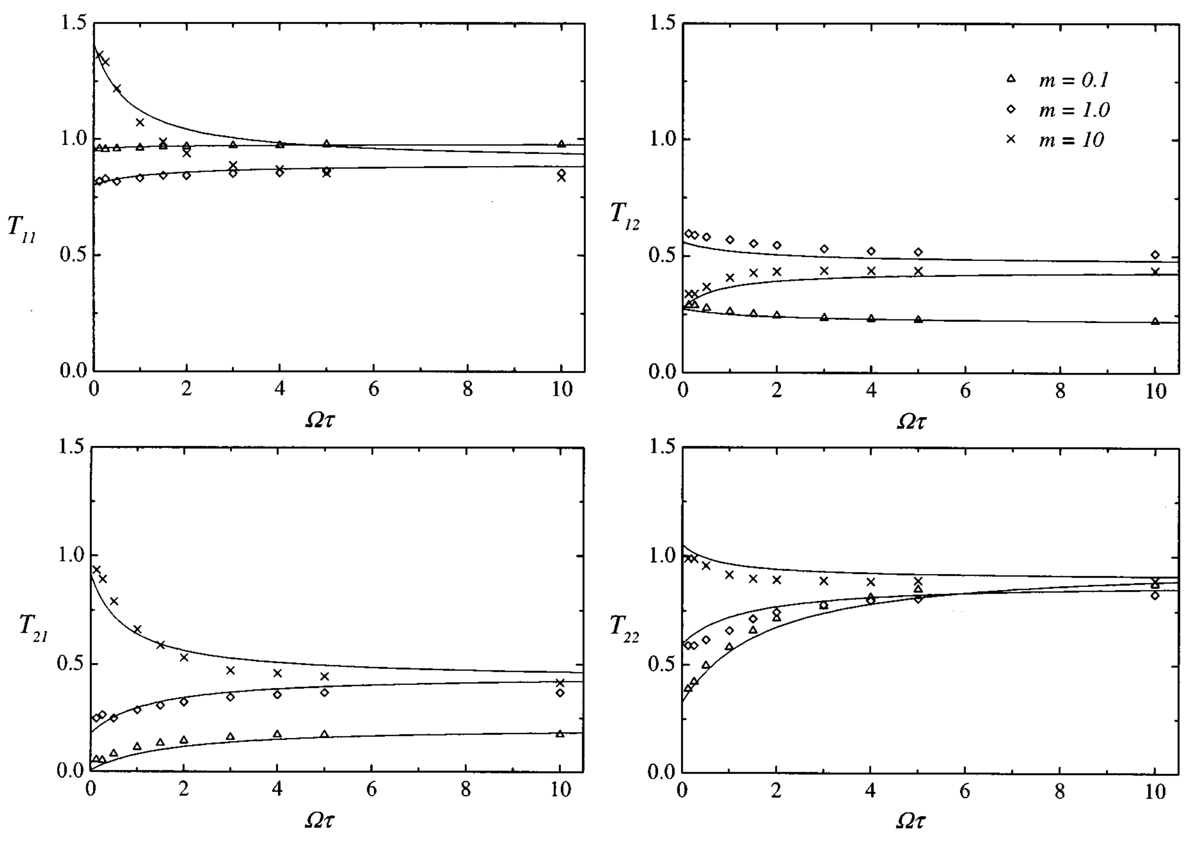

FIG. 2. Measured and calculated (solid lines) matrix elements for input beam intensity ratios of $m=0.1$, 1, and 10 for $\Gamma l \simeq 2.2$. The total power of the incident beams was 6.5 mW, and the time constant was 0.25 s.

It is seen that there is reasonable agreement between the theory and the measurements. The frequency dependence is most pronounced at low frequencies, and the matrix elements are close to their asymptotic values at $\Omega\tau \sim 10$. The slopes of the curves for $m=0.1$ and $m=1$ are of opposite sign than that for $m=10$. The sign changes when $m=e^{\Gamma l/2}$ ($\simeq 3$ for our experimental values) at which value the matrix elements are frequency independent. The variation of the matrix elements measured at the higher intensity ($P=55$ mW-not shown) was similar, although the higher intensity gave a slightly higher coupling constant, and also more pronounced beam fanning.

The matrix elements T_{11} and T_{22} describe the effective transmission of the fluctuations initially in the signal and pump beams. Matrix elements T_{12} and T_{21} describe cross coupling of fluctuations from pump to signal and signal to pump, respectively. The variation of the matrix elements with frequency leads to a frequency dependent redistribution of fluctuations between the beams. For example, as seen in Fig. 2, T_{12} falls at low frequencies for $m=10$. This implies that low-frequency fluctuations are suppressed when the signal beam is amplified. The ''missing'' fluctuations remain in the pump beam since T_{22} rises at low frequencies for $m=10$. In other words, amplification of a weak signal by a strong pump beam is accompanied by filtering low-frequency pump noise. Note that the beam for which noise is reduced or enhanced can be selected by varying the intensity ratio since $m-e^{\Gamma l/2}>0$ gives noise reduction in the signal beam, while $m-e^{\Gamma l/2}<0$ gives noise reduction in the pump beam.

In summary, the matrix elements describing the transfer of fluctuations in photorefractive two-beam coupling were measured and found to agree with a perturbative treatment of the interaction. The transfer is frequency dependent at low frequencies, and approaches a constant value when the fluctuation frequency exceeds several inverse time constants. In the low frequency regime amplification with pump noise, reduction is possible.

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[1] N. Kukhtarev, V. Markov, and S. Odulov, Opt. Commun. **23**, 338 (1977).
[2] L. Solymar and J. M. Heaton, Opt. Commun. **51**, 76 (1984).
[3] M. Cronin-Golomb, in *Technical Digest of Topical Meeting on Photorefractive Materials, Effects, and Devices* (Optical Society of America, Washington, DC, 1987), p. 142.
[4] G. C. Papen, B. E. A. Saleh, and J. A. Tataronis, J. Opt. Soc. Am. B **5**, 1763 (1988).
[5] F. Vachss, in *Technical Digest of Topical Meeting on Photorefractive Materials, Effects, and Devices* (Optical Society of America, Washington, DC, 1990), p. 140.
[6] M. Horowitz, D. Kligler, and B. Fischer, J. Opt. Soc. Am. B **8**, 2204 (1991).
[7] D. Z. Anderson and J. Feinberg, IEEE J. Quantum Electron. **25**, 635 (1989).
[8] A. Hermanns, C. Benkert, D. M. Lininger, and D. Z. Anderson, IEEE J. Quantum Electron. **28**, 750 (1992).
[9] M. Saffman, Ph.D. thesis, University of Colorado, 1994.
[10] A. A. Zozulya, M. Saffman, and D. Z. Anderson, J. Opt. Soc. Am. B **12**, 1036 (1995).
[11] F. Davidson, L. Boutsikaris, and M. Krainak, Opt. Lett. **13**, 506 (1988).
[12] T. J. Hall, M. A. Fiddy, and M. S. Ner, Opt. Lett. **5**, 485 (1980).
[13] G. Hamel de Monchenault and J. P. Huignard, J. Appl. Phys. **63**, 624 (1988).
[14] F. M. Davidson and L. Boutsikaris, Opt. Eng. (Bellingham) **29**, 369 (1990); F. Davidson, C. Field, and X. Sun, IEEE Photonics Technol. Lett. **4**, 1295 (1992).
[15] L.-A. de Montmorillon, I. Biaggio, P. Delaye, J.-C. Launay, and G. Roosen, Opt. Commun. **129**, 293 (1996).