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Break-up of two-dimensional bright spatial solitons due to transverse modulation instability

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Abstract. – We present the theory and the experimental observation of break-up of two-dimensional bright spatial solitons propagating in a three-dimensional bulk photorefractive nonlinear medium due to transverse modulation instability.

Solitary wave (self-bound) solutions of nonlinear propagation equations are a fascinating topic of nonlinear dynamics. The analysis of their stability and nonlinear evolution properties is one of the most crucial parts of the problem of self-trapping of optical beams. Much research has been devoted to this topic in connection with the question of the existence of optical solitons in media with Kerr-type nonlinearity (see, *e.g.*, ref. [1]-[8]). In this analysis it is important to understand the relationship between a particular solitary solution and possibly more general equations from which it has been derived. Despite an exact mathematical analogy between self-bound solutions their evolution may turn out to be completely different in physically different situations. Consider, *e.g.*, a two-dimensional solitary wave solution following from the two-dimensional basic set of equations (a pulse propagating in a single-mode fiber). This solution is a soliton since it can experience perturbations of only a certain limited class and is stable with respect to these perturbations [2]. The same two-dimensional solitary solution viewed in the context of, say, a cw beam propagating in a bulk medium is unstable [3]. The reason for this is that the same solution now belongs to a subclass of all possible solutions and has lower dimension $(1+1)$ than the dimension $(2+1)$ of the basic set of equations. It contains a "hidden" homogeneous coordinate (is independent of this coordinate) and is now subject to a broader class of perturbations. Of particular importance are perturbations that affect this hidden coordinate. The corresponding instability is called a transverse modulation instability. It goes back to papers by Bespalov and Talanov [4] and Benjamin and Feir [5] who discussed its manifestations for a homogeneous (plane-wave) ground state. Analyses of stability of two-dimensional self-bound solutions in a Kerr medium have been carried out in [7]-[8].

In this paper we analyze the evolution of two-dimensional bright spatial solitons that propagate in a three-dimensional bulk nonlinear photorefractive medium (the theoretical analysis

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of this paper has been reported in ref. [9]). We show that they are destroyed because of the symmetry breaking and the appearance of a spatial structure along the hidden homogeneous coordinate. We present experimental demonstrations of the break-up and the dependence of the instability on the parameters of the problem. Observations indicative of this instability for the case of solitary waves behind a ship were reported in [10]. The present work is, to our knowledge, the first experimental demonstration of the transverse modulation instability of two-dimensional solitary waves in any nonlinear optical medium.

We shall describe propagation of an optical beam $B(\mathbf{r})$ in a photorefractive medium in the presence of an externally applied electric field and/or photogalvanic nonlinearity by the set of equations [9], [11]

$$\left[\frac{\partial}{\partial x} - \frac{i}{2}\nabla^2\right] B(\mathbf{r}) = i\frac{\partial\varphi}{\partial z} B(\mathbf{r}) , \qquad (1a)$$

$$\nabla^2\varphi + \nabla \ln(1+|B|^2)\cdot\nabla\varphi = \frac{\partial}{\partial z}\ln(1+|B|^2) . \qquad (1b)$$

The differential operator ∇ acts on coordinates y and z perpendicular to the direction of propagation of the beam x. The dimensionless coordinates (x, y, z) are connected to the physical coordinates (x', y', z') by the relations $x = \alpha x'$ and $(y, z) = \sqrt{k\alpha}(y', z')$, where $\alpha = 2\gamma(E_{\mathrm{ext}} + E_{\mathrm{ph}})/\tilde{E}$. Here k is the wave number of electromagnetic radiation in the medium, φ is the electrostatic potential induced by the beam, 2γ is the intensity coupling constant, E_{ph} is the amplitude of the photogalvanic field, E_{ext} is the amplitude of the external field far from the beam, and $\tilde{E}$ is the characteristic value of the internal electrostatic field, governed by the material parameters of the medium. Both E_{ext} and E_{ph} are assumed to be directed along the z-coordinate. The electromagnetic intensity $|B(\mathbf{r})|^2$ is measured in units of the saturation intensity (also called dark intensity) I_{d}. The terms responsible for self-bending of the beam (fanning) have been artificially removed from eqs. (1). The boundary conditions for the potential φ are $\nabla\varphi(\mathbf{r} \to \infty) \to 0$.

If all functions in the system of equations (1) are assumed to depend only on the transverse coordinate z, the photorefractive nonlinearity becomes identical to the saturable Kerr nonlinearity. Equation ($1\,b$) can be integrated and eq. ($1a$) is recast in the form

$$\left[\frac{\partial}{\partial x} - \frac{i}{2}\frac{\partial^2}{\partial z^2}\right] B(x,z) = i\frac{|B|^2}{1+|B|^2} B . \qquad (2)$$

Equation (2) has a solitary wave solution $B(x, z) = b(z)\exp[i\mathit{\Gamma}_0 x]$, where $\mathit{\Gamma}_0 = 1 - b_{\mathrm{m}}^{-2}\ln(1+b_{\mathrm{m}}^2)$ is a propagation constant and $b_{\mathrm{m}} = b(0)$ is the maximum value of the amplitude $b(z)$, governed by the relation

$$(\mathrm{d}b/\mathrm{d}z)^2 = 2\left[\ln(1+b^2) - (b^2/b_{\mathrm{m}}^2)\ln(1+b_{\mathrm{m}}^2)\right] . \qquad (3)$$

The solitary solution (3) was first obtained in ref. [12], and discussed later in the context of a photorefractive nonlinearity in ref. [13].

To study stability of the solution (3) in three dimensions consider an electromagnetic field with a superimposed periodic modulation in y, $B(x, y, z) = [b(z) + \delta b(x, z)\sin(k_y y)]\exp[\mathit{\Gamma}_0 x]$, where $b(z)$ is the two-dimensional solution (3), k_y is an arbitrary transverse wave number, and δb is a small complex-valued perturbation function. The linearized system of equations for the perturbation is of the form (cf. [7])

$$\left[\frac{\partial}{\partial x} - \frac{i}{2}\frac{\partial^2}{\partial z^2}\right]\delta b = i\left(\frac{b^2}{1+b^2}\right)\delta b - i\left(\mathit{\Gamma}_0 + \frac{1}{2}k_y^2\right)\delta b + ib\delta\varphi , \qquad (4a)$$

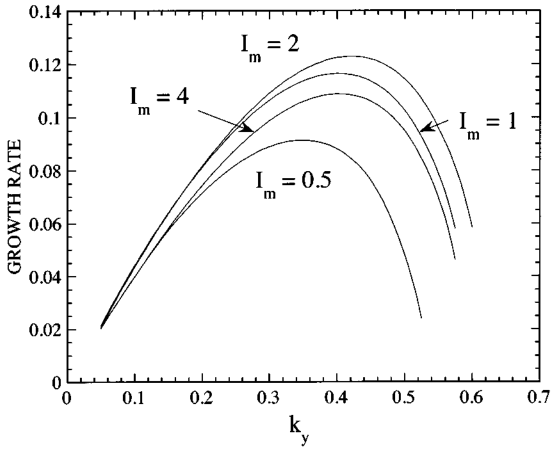

Fig. 1.

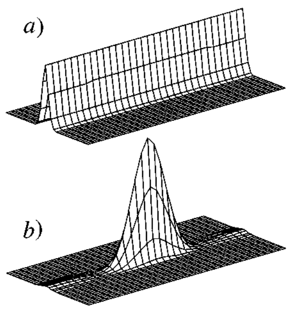

Fig. 2.

Fig. 1. – Modulation instability growth rate Γ as a function of the transverse wave number k_y for the bright solitary wave solution (3).

Fig. 2. – Initial intensity distribution of the solution (3) (a)) and the result of its modulation instability at $x \approx 58$ (b)).

$$\frac{\partial^2}{\partial z^2}\delta\varphi - k_y^2\delta\varphi + \frac{\partial}{\partial z}\left[\delta\varphi\,\frac{\partial}{\partial z}\ln(1+b^2)\right] - \frac{\partial}{\partial z}\left[\frac{1+b_{\rm m}^2}{1+b^2}\frac{\partial}{\partial z}\frac{b(\delta b+\delta b^*)}{(1+b^2)}\right] = 0. \tag{4b}$$

Solutions of eqs. (4) with exponential dependence on the coordinate x, $\delta b \propto \exp[\Gamma x]$ are eigenmodes that propagate without changing their spatial structure, and Γ is the corresponding eigenvalue. The eigenvalues may be either purely real or purely imaginary. Positive real values of Γ correspond to growing modes; their presence means that the ground-state homogeneous solution is unstable. The eigenmodes and eigenvalues (growth rates) Γ have been found numerically by propagating an arbitrary initial perturbation and calculating its growth rate. Figure 1 shows the instability growth rate Γ for the solution (3) as a function of the transverse wave number k_y for different values of its intensity in the center $I_{\rm m} = b^2(z=0)$. Note that the growth rates reach a maximum for a finite value $I_{\rm m}$ of the ratio of beam intensity to saturation intensity. Figure 1 demonstrates that the solution (3) is unstable in a certain range of transverse wave numbers k_y. The characteristic spatial scale of exponentially growing perturbations along the y-axis is comparable to the characteristic size of the ground-state soliton along the z-direction.

When the perturbation eigenmode grows, the initially homogeneous solution (3) breaks down into a set of pulses (filaments) along the y-axis. To study the nonlinear stage of the modulation instability, we solved eq. (1) numerically with periodic boundary conditions in y. The period of the calculation window along the y-coordinate l_y was chosen to correspond to one period of the fastest-growing mode wave number $k_y = k_y^*$ for a given value of $I_{\rm m}$ ($l_y = 2\pi/k_y^*$). The input boundary conditions corresponded to the self-bound two-dimensional solution (3) with small superimposed modulation $B(0,y,z) = b(z) + \epsilon f(z)\sin(k_y^* y)$, where $f(z)$ is an (arbitrary) even function of z of the order of unity decaying at infinity and ϵ is a small constant. For the results below ϵ was taken to be about 10^{-3}.

Figures 2 a), b) show the distribution of the beam intensity at the input to the medium $x = 0$ (a)) and for the value of the propagation coordinate $x \approx 58$ (b)). The initially

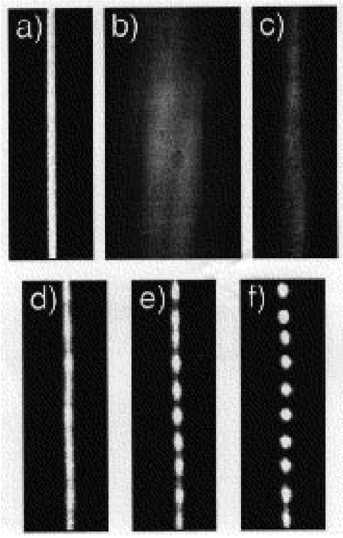

Fig. 3.

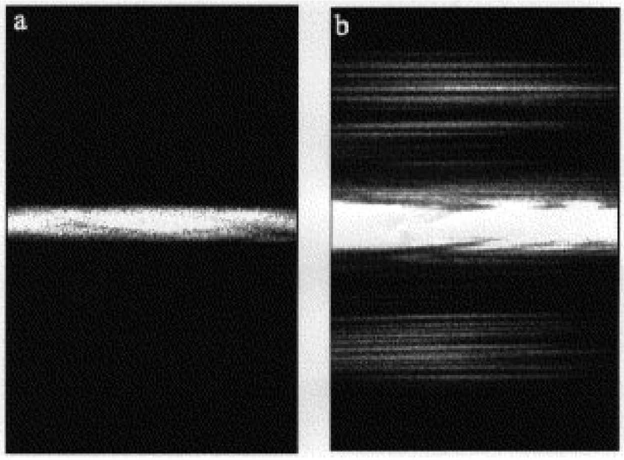

Fig. 4.

Fig. 3. – Input ($V = 0$) (*a*)) and output (*b*), *c*), *d*), *e*), *f*)) near-field intensity distributions for applied voltage equal to 0 V (*b*)), 600 V (*c*)), 1000 V (*d*)), 1560 V (*e*)), and 2200 V (*f*)).

Fig. 4. – Far-field intensity distributions for zero (*a*)) and nonzero (*b*)) values of the saturation intensity.

homogeneous ground state has collapsed into a sequence of pulses (only one period is shown). The calculations show that the intensity of the optical field in the nonlinear regime as a function of the propagation distance x oscillates between some varying minimum and maximum values. The calculations were repeated for a range of input beam widths up to 50% larger and smaller than the self-bound solution (3). In all cases the transverse modulation instability was seen.

In the experiments a 10 mW beam from a He-Ne laser ($\lambda = 0.6328$ μm) was passed through a variable beam splitter and a system of two cylindrical lenses. By adjusting the relative positions of the lenses the size of the elliptical beam waist could be controlled. The beam was directed into a photorefractive crystal of SBN:60, lightly doped with 0.002% by weight Ce. The beam propagated in the horizontal plane perpendicular to the crystal $\hat{c}$-axis, and was polarized in the horizontal plane along the $\hat{c}$-axis to take advantage of the largest component of the electro-optic tensor of SBN. The crystal measured 10 mm along the direction of propagation, and was 9 mm wide along the $\hat{c}$-axis. The elliptical beam waist measuring about 15 μm in the horizontal direction by 2 mm in the vertical direction was placed about 0.2 mm in front of the crystal, so that the input beam in the crystal was diverging. A variable dc voltage was applied along the $\hat{c}$-axis to control the value of nonlinear coupling, and the effective saturation intensity was varied by illuminating the crystal from above with incoherent white light. Images of the intensity distribution were recorded with a CCD camera.

Figure 3 shows the near-field distributions of the input (*a*)) and the output (*b*), *c*), *d*), *e*), *f*)) beam for different values of the applied voltage (different values of the nonlinearity) and a fixed level of incoherent illumination several times weaker than the beam intensity. All pictures were recorded under steady-state conditions. Figure 3 *b*) shows the diffractive spreading of the output beam for zero applied voltage (zero nonlinearity). As the nonlinearity increases the beam starts to self-focus (*c*)) forming a self-trapped channel of light (*d*)). Increasing

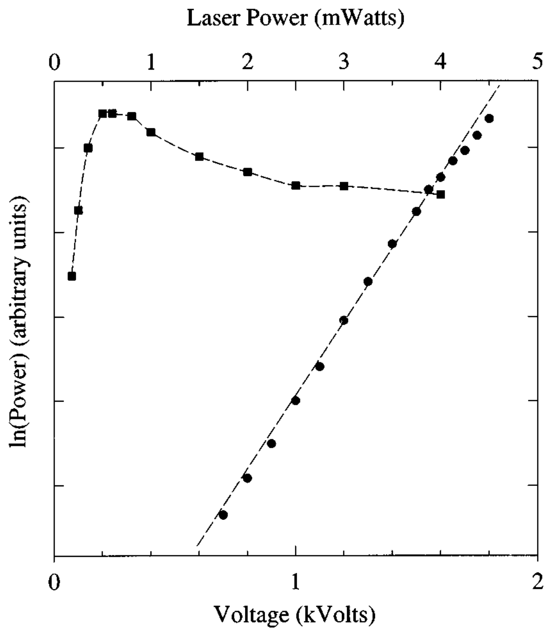

Fig. 5. – Total power of the amplified noise as a function of the laser power with 1.8 kV applied voltage (upper curve, squares), and as a function of the applied voltage for a laser power of 0.3 mW (lower curve, circles). The maximum value of the integrated noise power corresponds to about 10% of the incident-beam power. The dashed lines have been drawn as visual aids.

the nonlinearity unavoidably turns on the modulation instability. In addition to the two-dimensional input profile the electromagnetic field in the medium also contains some small amount of noise that can be decomposed in a set of Fourier harmonics with different transverse wave numbers k_y. In a linear regime the intensity of each harmonic grows with the propagation distance as $\epsilon \exp[2\Gamma x]$, where $\Gamma(k_y)$ is the harmonics' growth rate (see fig. 1) and ϵ is its initial intensity. For larger nonlinearities the amplified noise grows to the extent of becoming noticeable (*e*)) and the two-dimensional self-trapped beam breaks up into a periodic sequence of filaments (*f*)). It should be emphasized that no artificial seeding was added to the input beam. The instability developed from the natural level of noise present on the beam and/or in the crystal. The period of the modulation in fig. 3 *f*) is equal to about 40 μm or $k_y/k \approx 0.015$. To compare with the dimensionless values used in fig. 1, we used a value of 240 pm/V for the electro-optic coefficient of SBN in this geometry, which gives $\alpha \approx 29$ cm^{-1} at the voltage used in fig. 3 *f*). The measured value of k_y/k thus corresponds to a dimensionless value of about 0.3, which is in good correspondence with the functional dependence shown in fig. 1. As the voltage was increased the center of the focused channel shown in frames *d*) to *f*) was also displaced along the $\hat{c}$-axis by about 2 beam diameters due to photorefractive self-bending [14]. The experiments were repeated with a range of input beam widths up to twice larger and smaller than that used in fig. 3. The transverse instability was observed in all cases.

The symmetry breaking and the generation of the spatial structure seen in fig. 3 *f*) are manifested in the far-field (Fourier plane) by the appearance of new components in the Fourier spectrum due to the amplification of noise. This is illustrated in fig. 4 which shows the far-field distribution of the output beam for zero (*a*)) and nonzero (*b*)) external illumination, with the same value of applied voltage.

The level of the nonlinearity necessary for the observation of the instability depends on the value of the saturation intensity I_{d} with respect to the intensity of the beam I_{beam}. Figure 5 (upper curve) shows the dependence of the integrated power of the amplified noise on this ratio by keeping the same intensity of incoherent illumination and changing the laser intensity. The power had a maximum corresponding to a saturation intensity several times smaller than the intensity of the beam. Figure 5 (lower curve) shows the dependence of the integrated power of the amplified noise on the applied voltage for fixed laser intensity. Except for the highest voltages, where a slight saturation is evident, the growth of the amplified noise is described well by an exponential dependence on applied voltage.

Under transient conditions the transverse modulation instability was clearly seen both with and without additional external illumination. In steady state the instability has been observed only for nonzero external illumination (see fig. 4), since the natural level of the saturation intensity in the crystal was too small.

In summary, we have presented the theory and the first experimental demostration of break-up of two-dimensional bright spatial solitons propagating in a three-dimensional bulk nonlinear medium due to transverse modulation instability.

Additional Remark.

The corresponding instability for a dark spatial soliton has been reported recently [15].

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REFERENCES

[1] Rasmussen J. J. and Rypdal K., *Phys. Scr.*, **33** (1986) 481.
[2] Vakhitov N. G. and Kolokolov A. A., *Izv. Vyssh. Uchebn. Zaved. Radiofiz.*, **16** (1973) 1020 (*Sov. Radiophys.*, **9** (1976) 261).
[3] Zakharov V. E. and Rubenchik A. M., *Zh. Èksp. Teor. Fiz.*, **65** (1973) 997 (*Sov. Phys. JETP*, **38** (1974) 494).
[4] Bespalov V. I. and Talanov V. I., *Pis'ma Zh. Èksp. Teor. Fiz.*, **3** (1966) 471 (*JETP Lett.*, **3** (1966) 307).
[5] Benjamin T. B. and Feir J. E., *J. Fluid Mech.*, **27** (1967) 417.
[6] Akhmediev N. N., Heatley D. R., Stegeman G. I. and Wright E. M., *Phys. Rev. Lett.*, **65** (1990) 1423.
[7] Akhmediev N. N., Korneev V. I. and Nabiev R. F., *Opt. Lett.*, **17** (1992) 393.
[8] McDonald G. S., Syed K. S. and Firth W. J., *Opt. Commun.*, **95** (1993) 281.
[9] Zozulya A. A. and Anderson D. Z., in *Proceedings of the International Conference on Photorefractive Materials, Effects and Devices, Estes Park, Colorado*, 1995, edited by D. Z. Anderson and J. Feinberg (unpublished), p. 399.
[10] Brown E. D. *et al.*, *J. Fluid Mech.*, **204** (1989) 263.
[11] Zozulya A. A. and Anderson D. Z., *Phys. Rev. A*, **51** (1995) 1520.
[12] Gatz S. and Herrmann J., *J. Opt. Soc. Am. B*, **8** (1991) 2296.
[13] Valley G. C. *et al.*, *Phys. Rev. A*, **50** (1994) R4457; Segev M. *et al.*, *Phys. Rev. Lett.*, **73** (1994) 3211.
[14] Feinberg J., *J. Opt. Soc. Am.*, **72** (1982) 46.
[15] Mamaev A. V., Saffman M. and Zozulya A. A., *Phys. Rev. Lett.*, **76** (1996) 2262.