Manipulating the information carried by an optical beam with reflexive photorefractive beam coupling

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A two-beam coupling interaction of an information-bearing beam with a copy of itself in a photorefractive medium changes the spatiotemporal nature of the beam. We analyze the reflexive coupling of an information-bearing beam carrying two or more spatiotemporal signals, using a one-dimensional model. We find that the statistical information content of the beam can be increased or decreased by use of reflexive coupling. In particular, we show that the reflexive interaction can be used to selectively enhance a given signal over others on the basis of intensity and that, for example, a pair of spatially overlapping but temporally independent signals can be made spatially orthogonal.

1. INTRODUCTION

In this paper we consider the signal-processing characteristics of the photorefractive two-beam coupling interaction shown in Fig. 1, in which an information-bearing light beam interacts with a copy of itself. We refer to the interaction geometry shown in Fig. 1 as reflexive coupling. We assume that the information carried by the beam consists of two or more spatially and temporally distinct signals. For example, the beam may consist of several images that appear at different times, or of images that are imposed upon distinct carrier frequencies, or of a collection of temporal signals carried by different spatial patterns.

The basic idea is that the refractive-index grating formed in a photorefractive medium by the reflexive interaction of an optical beam containing a number of temporal signals with itself consists of a superposition of components, each one due to the self-interaction of just one of the temporal signals. Even though each temporal signal interacts only with itself, there is an indirect coupling between different temporal signals that is due to shared spatial components of the photorefractively induced index grating. The indirect coupling through shared gratings[1] allows signals to influence each other even though they may have carrier frequencies separated by more than the response bandwidth of the photorefractive medium or are present at different times. The result of the indirect coupling is that the fractional intensity and the spatial overlap of different temporal signals are modified at the output ports. The dispersion of the fractional intensities of the temporal signals carried by the optical beam determines the statistical information content of the beam. The statistical information is not conserved by reflexive interaction, and, depending on the choice of output port, the statistical information may be selectively increased or decreased. It is also possible to selectively manipulate individual signals. In particular, as is shown in what follows, it is possible to spatially orthogonalize a pair of temporal signals that initially have a finite spatial overlap.

A special case of practical importance arises when the temporal signals are also spatially orthogonal. The spatially orthogonal signals interact (are Bragg matched) only with their own gratings and do not interact (are Bragg mismatched) with all other grating components. On the other hand the modulation depth of each grating component is reduced by the presence of additional spatially orthogonal signals. Thus, if one signal is initially strongest, it also writes the strongest grating and experiences the largest energy coupling in the geometry of Fig. 1. By utilizing this effect in several different ways we can enhance strong signals, extract weak signals from a strong background, or equalize the intensity of several unequal signals. This type of intensity-based manipulation was used in a photorefractive frequency demultiplexer[2] and a photorefractive feature extractor[3] as a contrast enhancer to force the oscillation of a single frequency in a photorefractive ring resonator. It has also been used in a photorefractive signal extractor circuit to extract weak optical signals out of a multisignal beam. A similar geometry was used for radio-frequency notch filtering in the research reported in Ref. 4. By allowing for a path-length difference in the two arms of Fig. 1 it is also possible to obtain frequency narrowing of a laser line, as has been demonstrated in Ref. 5. Photorefractive two-beam coupling of partially coherent light has also been studied in several recent publications.[6–8]

We analyze reflexive photorefractive coupling of multisignal beams, using a one-dimensional model, in Section 2 below. The manipulation of spatially orthogonal signals based on their intensity is described in Section 3, and modification of the spatial overlap between signals is analyzed in Section 4.

2. THEORY

Consider an optical beam consisting of a superposition of temporally distinct signals

$$|E(z,\, t)\rangle = \sum_e e(z,\, t)|e\rangle \exp[i(kz - \omega t)]\,. \qquad (1)$$

We find it convenient to use Dirac state vector notation

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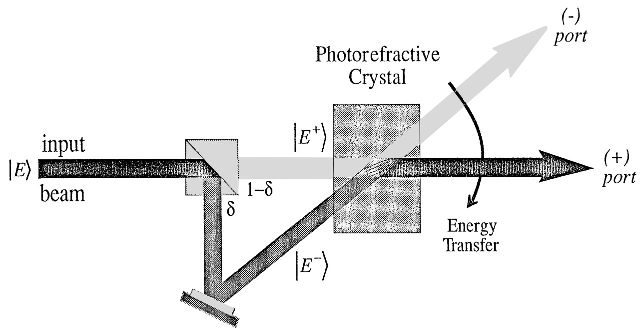

Fig. 1. Reflexive coupling geometry.

to describe a general one-dimensional transverse distribution of the optical beam. The spatial components $\{|e\rangle\}$ are taken to be normalized to unity: $\langle e|e\rangle = 1$, although the spatial profiles may overlap so that in general $\langle e|e'\rangle \neq 0$. $e(z, t)$ is a slowly varying amplitude with both a temporal dependence and a spatial dependence that is due to the photorefractive coupling, k and ω are the wave vector and the frequency, respectively, and all signals are assumed to have the same polarization so the electric field may be written as a scalar quantity. From now on we will drop the rapidly varying exponential factor for notational simplicity.

The response of the photorefractive medium will be used to quantify the notion of temporally distinct signals. We characterize signals as temporally distinct if they are temporally orthogonal as measured by the photorefractive medium, i.e., if

$$\rho_{ee'}(z, t) \equiv (1/\tau)\int_{-\infty}^{t} \mathrm{d}t' \exp[-(t - t')/\tau] \times e(z, t')e'^{*}(z, t') = I_e(z, t)\delta_{ee'}\,, \qquad (2)$$

then we say that signals $e(z, t)$ and $e'(z, t)$ are temporally mutually orthogonal, and the interference of two different signals does not induce a material response in the photorefractive medium. In Eq. (2) τ is the characteristic photorefractive response time and $I_e(z, t)$ is proportional to the exponentially weighted time-averaged optical intensity. We further assume that $I_e(0, t)$ is nominally independent of time so that we may make the replacement $I_e(0, t) = I_e = |e|^2$. Signals may be temporally orthogonal as a result, for example, of their being present at different times or having different carrier frequencies. In photorefractive media τ scales as the inverse of the total incident intensity ($\tau \sim 1/\sum_e I_e$). Thus, for example, the temporal overlap of sinusoidal signals with a small frequency difference decreases as the intensity is lowered. Any pair of mutually temporally incoherent signals is also temporally orthogonal as measured by the photorefractive medium, although mutually temporally coherent signals may or may not be temporally orthogonal. The beam splitter in Fig. 1 separates the incident field into two components:

$$|E^+(z, t)\rangle = \sum_e e^+(z, t)|e^+\rangle\,, \qquad (3a)$$

$$|E^-(z, t)\rangle = \sum_e e^-(z, t)|e^-\rangle\,, \qquad (3b)$$

where $e^+(0, t) = \sqrt{1 - \delta}\, e(0, t)$, $e^-(0, t) = \sqrt{\delta}\, e(0, t)$, and δ is the fractional intensity diverted by the beam splitter. The photorefractive grating formed by the interference of the (+) and (−) components is given by

$$\hat{G}(z, t) = \frac{\Gamma}{2I_{\text{tot}}} \sum_{e,e'} \rho_{e^+e'^-}(z, t)|e^+\rangle\langle e'^-| = \frac{\Gamma}{2I_{\text{tot}}} \sum_{e} \rho_{e^+e^-}(z, t)|e^+\rangle\langle e^-|\,, \qquad (4)$$

where the second equality follows from the temporal orthogonality of the signals, Γ is the photorefractive coupling coefficient, and $I_{\text{tot}} = \sum_e I_e$ is the total time-averaged intensity incident upon the photorefractive medium. The total grating formed in the photorefractive medium, as given by Eq. (4), is a superposition of possibly overlapping gratings, each due to the interference of a single temporal signal with itself. There are several approximations implicit in the form of Eq. (4). We have assumed that the angular spread of each beam is much less than the angle between the intersecting beams such that there is no direct coupling between spatial modes within each beam and that the interbeam coupling constant is the same for all modes. Any relative phase between $e^+(0, t)$ and $e^-(0, t)$ that results from unequal propagation lengths in the reflexive paths is assumed negligible. The presence of such phase shifts leads to spatially mismatched grating components for different temporal signals, even though the spatial modes of the signals are identical. This effect is useful, for example, for laser frequency narrowing[5] but will not be considered further here. Finally, the $\{e(z, t)\}$ are taken to vary slowly enough that there is no Bragg mismatching as a result of different temporal frequencies. For visible wavelengths, and for centimeter-sized crystals, the temporal bandwidth can be as large as several gigahertz before Bragg mismatching becomes significant. The spatial evolution of the fields that is due to the photorefractive grating is described by

$$\frac{\partial}{\partial z}|E^+(z, t)\rangle = \hat{G}(z, t)|E^-(z, t)\rangle\,, \qquad (5a)$$

$$\frac{\partial}{\partial z}|E^-(z, t)\rangle = -\hat{G}^\dagger(z, t)|E^+(z, t)\rangle\,. \qquad (5b)$$

Equations (4) and (5) have a general solution, which is considered in Section 4 below. When the temporal signals are also spatially orthogonal, such that $\langle e|e'\rangle = \delta_{ee'}$, we can project Eqs. (4) and (5) onto the spatial modes $\{|e\rangle\}$ by integrating over the transverse coordinate and obtain the simplified equations

$$\frac{\partial}{\partial z} e^+(z, t) = G_e(z, t)e^-(z, t)\,, \qquad (6a)$$

$$\frac{\partial}{\partial z} e^-(z, t) = G_e^{*}(z, t)e^+(z, t)\,, \qquad (6b)$$

$$G_e(z, t) = \frac{\Gamma}{2I_{\text{tot}}} e^+(z, t)e^{-*}(z, t)\,. \qquad (6c)$$

In writing Eqs. (6) we have made some assumptions about the structure of the spatial modes $|e\rangle$. Suppose that $E(z, t)\rangle = e_1(z, t)|e_1\rangle + e_2(z, t)|e_2\rangle$; then direct application of Eqs. (4) and (5a) implies that the evolution of signal 1 in the (+) arm of the reflexive interaction as described by Eq. (6a) is also dependent on a term proportional to $|e_2{}^+\rangle\langle e_2{}^-|e_1{}^-\rangle$. This term is not present on the right-hand side of Eq. (6a) because we have assumed that $\langle e_2|e_1\rangle = \delta_{e_2e_1}$. However, even though $\langle e_2|e_1\rangle = \delta_{e_2e_1}$ at the entrance face of the photorefractive medium we may find, depending on the specific structure of the modes, that $\langle e_2|e_1\rangle \neq \delta_{e_2e_1}$ inside the nonlinear medium. Indeed, it has been demonstrated that the mutual spatial coherence can be changed by the nonlinear interaction.[6–8] When the spatial modes are markedly different, for example, if they have no intensity overlap, then Eqs. (6) are an excellent approximation. In the one-dimensional model considered here it is assumed that spatial orthogonality is preserved by the photorefractive interaction, which allows the spatial evolution to be formulated in terms of the relatively simple coupled-mode equations (6) for the amplitudes $\{e^{\pm}\}$. It should also be emphasized that Eqs. (6) are applicable only when the beams fully cross inside the photorefractive medium, such that the entire transverse profile of each beam interacts with the entire transverse profile of the other, as is shown in Fig. 1. If this were not the case, spatially orthogonal signals would experience strong diffraction from each other's gratings. Since the beams do not cross in the direction perpendicular to the plane of Fig. 1 the spatial modes are assumed to be functions of a single transverse coordinate x only, which is taken to lie in the plane of the interacting beams. For beams with complex spatial structure, for example speckle patterns, diffraction provides effective mixing along the coordinate perpendicular to the plane of the beam crossing. In such cases signals with two-dimensional structure may be processed effectively with the interaction geometry shown in Fig. 1.

Equations (6) are equivalent to the standard equations of photorefractive two-beam coupling[9] but with a reduced coupling coefficient $\Gamma \rightarrow \Gamma I_e/I_{\mathrm{tot}}$ that is due to the dependence of the photorefractive response on the modulation depth of the interference of two beams. In the case of purely real Γ, which results in energy transfer between the interacting beams but no phase transfer, the solutions take the well-known form

$$I_e^{\pm}(l) = I_e^{\pm}(0)\,\frac{1 + r^{\pm 1}}{1 + r^{\pm 1}\exp(\mp \Gamma l I_e/I_{\mathrm{tot}})}\,, \tag{7}$$

where the beam-splitter ratio is $r = \delta/(1-\delta)$ and l is the length of the interaction region.

When the signals are temporally and spatially orthogonal they may be conveniently labeled by the fractional intensities $q_e = I_e/I_{\mathrm{tot}}$, where $\sum_e q_e = 1$. Solutions (7) may then be written in terms of the fractional intensities of the signals at each output port in the form

$$q_e^{\pm} = \frac{I_e^{\pm}(l)}{I^{\pm}(l)} = \frac{1}{Q^{\pm}}\,\frac{q_e}{1 + r^{\pm 1}\exp(\mp \Gamma l q_e)}\,, \tag{8a}$$

$$Q^{\pm} = \frac{I^{\pm}(l)}{I_{\mathrm{tot}}} = \sum_e \frac{q_e}{1 + r^{\pm 1}\exp(\mp \Gamma l q_e)}\,, \tag{8b}$$

where $q_e^{\pm}$ are the fractional intensities of each signal at the (+) and (−) output ports and $Q^{\pm}$ are the fractions of the total incident intensity present at each of the two output ports. The change in the fractional intensities of the signals at the output ports depends critically on the factor $r^{\pm 1}\exp(\mp \Gamma l q_e)$. When this factor is small the fractional intensity of the signal is enhanced, and when this factor is large the signal is suppressed.

Before proceeding to examine some particular cases it is instructive to rewrite solutions (8) in terms of statistical information. We have assumed that each temporally orthogonal signal in the incident beam is also spatially orthogonal. The statistical information content of the beam is therefore proportional to[10] $H = -\sum_e q_e \ln(q_e)$. The statistical information at the output ports is

$$\begin{aligned} H^{\pm} &= -\sum_e q_e^{\pm}\ln(q_e^{\pm}) \\ &= \ln Q^{\pm} - \frac{1}{Q^{\pm}}\sum_e \frac{q_e}{1 + r^{\pm 1}\exp(\mp \Gamma l q_e)} \\ &\quad \times \left[\frac{q_e}{1 + r^{\pm 1}\exp(\mp \Gamma l q_e)}\right], \end{aligned} \tag{9}$$

which may be larger or smaller than H. The variation of the information center at the (+) and (−) output ports is shown in Fig. 2 as a function of the adjustable interaction parameters r and Γl. For not-too-large coupling strengths the information content is increased at the (−) port and decreased at the (+) port. This implies that the signal amplitudes tend to become more closely balanced at the (−) port and more unequal at the (+) port. For very large coupling strengths all the energy is simply transferred to the (+) port with no discrimination between the signals, so that the information content at the (+) port tends toward the input information content. The information content at the (−) port tends to zero for very large coupling strengths, but this is not physically meaningful because the intensity becomes exponentially small.

3. INTENSITY-BASED SIGNAL PROCESSING

The reflexive interaction is useful for manipulating spatially orthogonal signals in a multisignal beam on the basis of their intensity. We describe three examples of signal manipulation in this section: beam cleanup by enhancing the strongest signal and suppressing all weaker signals, extracting a weak signal from a background of stronger signals, and equalization of unequal signals.

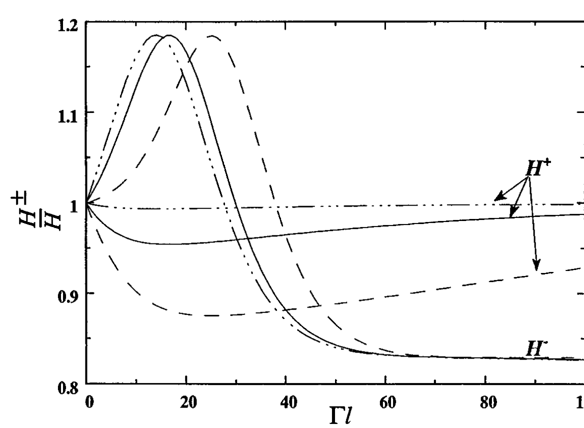

Fig. 2. Distribution of information at the two ports. The normalized output information is shown at the (+) and (−) ports for 10 input signals with $H = 1.94 = 0.84\ \ln(10)$ and —. . .—, $r = 0.1$; ———, $r = 1$; and – – – –, $r = 10$.

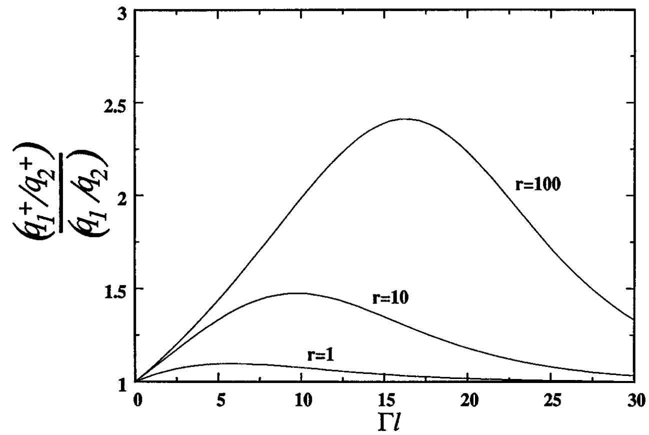

Fig. 3. Enhancement of the strongest input signal at the (+) output port for $q_1 = 0.26$, $q_2 = 0.185$, $q_1/q_2 = 1.41$, $N = 5$, and $r = 0.1, 1, 10$.

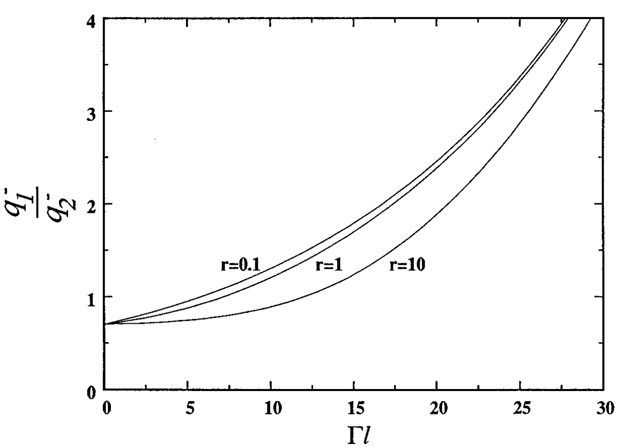

Fig. 4. Extraction of a weak signal at the (−) output port for $q_1 = 0.15$, $q_2 = 0.2125$, $q_1/q_2 = 0.71$, $N = 5$, and $r = 0.1, 1, 10$.

Consider a multisignal beam in which a particular signal e is stronger than all the others. The component of the total grating written by the term e^+e^{-*} is thus stronger than all the other grating components, and signal e will be transferred most efficiently from port (−) to port (+). The result is an increase in contrast between signal e and the other signals at port (+) compared with the input beam contrast. Under favorable conditions it is possible to clean up a multisignal beam such that close to 100% of the energy at the (+) port is due to single signal. It should be remembered that the other signals have not disappeared; their energy is present at the (−) output port. Simplified analytical expressions for the output contrast may be obtained by assuming particular input distributions. Consider, for example, a beam of N signals with fractional intensities q_1 and $q_2 = q_3 \ldots = q_N$. Application of Eqs. (8) then gives

$$\frac{q_1^+}{q_2^+} = \frac{q_1}{q_2}\,\frac{1 + r\,\exp(-\Gamma l q_2)}{1 + r\,\exp(-\Gamma l q_1)}, \tag{10}$$

and for $q_1 > q_2$ the contrast of signal 1 at the (+) port is always greater than the input contrast. The improvement in contrast ratio given by Eq. (10) is shown in Fig. 3 as a function of r and Γl.

To extract a weak signal from a background of stronger signals we instead use the (−) output port. Because the stronger signals write stronger gratings, their energy is more effectively transferred from the (−) to the (+) port. Under certain conditions what is left behind in port (−) consists mostly of one of the weaker signals. Assuming again a beam of N signals with fractional intensities q_1 and $q_2 = q_3 \ldots = q_N$, we find that

$$\frac{q_1^-}{q_2^-} = \frac{q_1}{q_2}\,\frac{1 + r^{-1}\,\exp(\Gamma l q_2)}{1 + r^{-1}\,\exp(\Gamma l q_1)}\,. \tag{11}$$

We may obtain $q_1^- > q_2^-$ even though $q_1 < q_2$ when the right-hand side of Eq. (11) is > 1. When $q_1 \ll q_2$ we have $q_2 \cong 1/(N - 1)$, and the condition on the coupling constant for inversion of the intensity ratio becomes

$$\Gamma l(N - 1)[\ln(1/q_1) - \ln(N - 1)]\,. \tag{12}$$

Thus one needs large coupling constants to extract very weak signals. This further motivates the use of photorefractive media for which coupling constants as high as 45 cm^{-1} have been measured.[11] When q_1 and q_2 are of comparable magnitude the necessary values of r and Γl may be found by numerical evaluation of Eq. (11). Some typical results are shown in Fig. 4.

More generally there may be N signals all of different intensities. We wish to extract any one of the N signals and make it the strongest signal at the (−) output port. Assume, for example, that the input signal intensities vary linearly: $q_n = \alpha n$ for $n = \{1, \ldots, N\}$. Equations (8) then give

$$q_n^- = \frac{1}{Q^-}\,\frac{\alpha n}{1 + r^{-1}\,\exp(\Gamma l \alpha n)}\,. \tag{13}$$

The results of using Eq. (13) to select any of N different signals are shown in Fig. 5 for fixed r and variable Γl. In practice Γl may be varied easily, for example, by adjustment of the intensity of an additional incoherent erasure beam that illuminates the photorefractive medium. Extraction of weak signals was demonstrated experimentally in Ref. 4. It can be seen that the achievable contrast for the intermediate-intensity signals is rather low. In principle this could be remedied by multiple reflexive

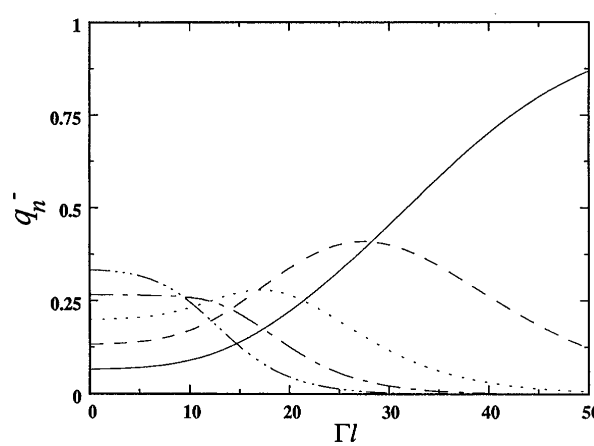

Fig. 5. Selective extraction of different-intensity signals by varying the coupling constant. The parameters are $r = 30$, $q_n = \alpha n$, and $N = 5$. The curves correspond to ———, $n = 1$; — — — —, $n = 2$; · · ·, $n = 3$; — — —, $n = 4$; and —· · ·—, $n = 5$.

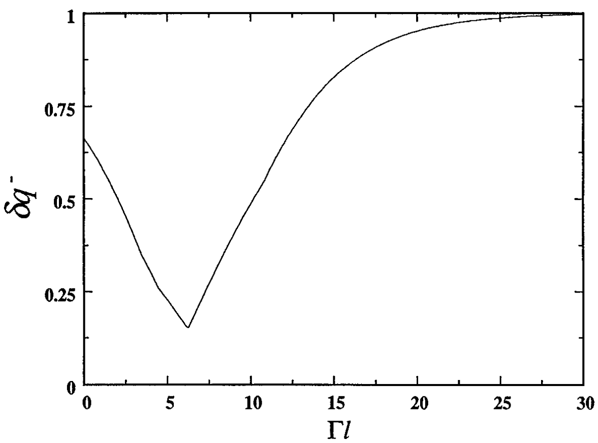

Fig. 6. Balancing of unequal-intensity signals. The parameters are $r = 0.1$, $q_n = \alpha n$, and $N = 5$.

stages, with the first stage extracting the desired weak signal using the (−) port and the next stage contrast enhancing using the (+) port. The practical limitation to such architectures arises from the inevitable loss of energy at each reflexive stage.

Finally, in some applications it may be desirable to equalize the intensities of a number of unequal signals. For example, the photorefractive device demonstrated in Refs. 2 and 3 requires that the input signals have equal intensities to within ~10%. By again using the (−) output port and choosing the values of r and Γl appropriately it is possible to balance closely a number of unequal input signals. The correct settings are most easily found numerically. An illustrative case for five input signals with linearly spaced intensities is shown in Fig. 6, where the fractional spread in $\{q_e^-\}$ defined by $\delta_q^- = (q_e^-|_{\max} - q_e^-|_{\min})/(q_e^-|_{\max} + q_e^-|_{\min})$ is plotted as a function of Γl. For $\Gamma l \sim 6.5$ the initial spread of $\{q_e\}$ characterized by $\delta q = 0.67$ is reduced to $\delta q^- = 0.15$.

4. SIGNALS WITH SPATIAL OVERLAP

We wish to understand what happens for an arbitrary field of spatially overlapping signals. In order to discriminate effectively between the temporal signals, for example, by use of an optical correlator or a self-organizing feature extractor,[2,3] it is useful first to orthogonalize them spatially. We need, therefore, to solve Eqs. (4) and (5) without assuming spatial orthogonality.

The solution of the problem entails finding a new basis for $|E\rangle$ in which both the temporal and the spatial components are orthogonal. We can then use the results of Section 2 to solve for the effects of reflexive coupling. It is perhaps easiest to see how to select the proper basis set by first projecting the field state vector onto an orthonormal basis $\{|o\rangle\}$:

$$|E(z, t)\rangle = \sum_o o(z, t)|o\rangle, \tag{14}$$

where, with $\langle o|o'\rangle = \delta_{oo'}$, the amplitudes are given by

$$o(z, t) = \sum_e e(z, t)\langle o \,|\, e\rangle. \tag{15}$$

The temporal overlap of the amplitudes in the new basis is given by

$$\begin{aligned}\rho_{oo'} &= \sum_{e,e'} \rho_{ee'}\langle o \,|\, e\rangle\langle e' \,|\, o'\rangle \\ &= \sum_e \rho_{ee}\langle o \,|\, e\rangle\langle e \,|\, o'\rangle \\ &= \sum_e I_e\langle o \,|\, e\rangle\langle e \,|\, o'\rangle.\end{aligned} \tag{16}$$

We can always diagonalize ρ with a unitary transformation U. With the unitary operator U in hand we are essentially back to the conditions of the previous sections in which the input signal is composed of N temporally and spatially orthogonal beams.

We find the effect of the reflexive interaction on a particular temporal signal by looking at the amplitude of the same temporal signal at one of the output ports:

$$\begin{aligned}e^\pm(l, t)|e^\pm(l)\rangle &= e^\pm(0, t)\sum_o \frac{o^\pm(l, t)}{o^\pm(0, t)}\langle o^\pm \,|\, e^\pm(0)\rangle \,|\, o^\pm\rangle \\ &= e^\pm(0, t)\sum_o g_o^\pm\langle o^\pm \,|\, e^\pm(0)\rangle \,|\, o^\pm\rangle.\end{aligned} \tag{17}$$

In Eq. (17) the basis set $\{|o\rangle\}$ is assumed chosen such that ρ defined by Eq. (16) is diagonal and

$$|g_o^\pm|^2 \equiv \frac{I_o^\pm(l)}{I_o^\pm(0)} = \frac{1 + r^{\pm 1}}{1 + r^{\pm 1}\exp(\mp\Gamma l q_o)} \tag{18}$$

from Eq. (7).

Clearly the signals change in both magnitude and direction in Hilbert space. It is because the signal vectors change direction that one can perform effectively a Gram–Schmidt orthogonalization of them, using a series of two-beam coupling interactions. One can do so provided that the signals form a linearly (spatially) independent set. In general, it takes $N - 1$ two-beam coupling interactions to orthogonalize N signals. The normalized spatial overlap of two output signals, $|e(l, t)\rangle$ and $|e'(l, t)\rangle$, is given by

$$h_{ee'}^\pm = \frac{\sum_o |g_o^\pm|^2 I_o^\pm(0)\langle o^\pm \,|\, e\rangle\langle e' \,|\, o^\pm\rangle}{\sum_o |g_o^\pm|^2 I_o^\pm(0)}, \tag{19}$$

and when $h_{ee'}^\pm = 0$ the output signals are spatially orthogonal. It is the (−) port, where the signals experience loss, that produces the orthogonalization.

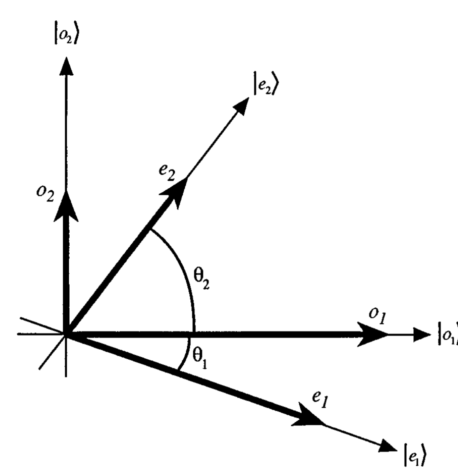

Fig. 7. Vectorial representation of spatially overlapping input signals.

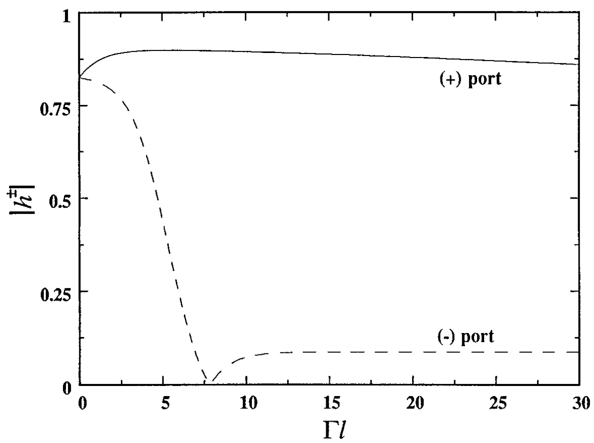

Fig. 8. Evolution of the normalized spatial overlap $h^{\pm}$ for $\theta_1 + \theta_2 = 0.6$ rad, $q_{e_1} = q_{e_2} = 0.5$, and $r = 10$.

Let us take the case of orthogonalization of two spatially overlapping signals,

$$|E(z, t)\rangle = e_1(z, t)|e_1\rangle + e_2(z, t)|e_2\rangle, \tag{20}$$

as a simple but illustrative example. Diagonalization of the matrix r defined in Eq. (16) for a 2×2 matrix is straightforward and simply requires that

$$\frac{I_{e_2}}{I_{e_1}} = -\frac{\langle e_1 | o_2\rangle\langle o_1 | e_1\rangle}{\langle e_2 | o_2\rangle\langle o_1 | e_2\rangle} = -\frac{\sin 2\theta}{\sin 2\theta_2}, \tag{21}$$

where θ_1 and θ_2 are the angles of $|e_1\rangle$ and $|e_2\rangle$ with respect to $|o_1\rangle$, as shown in Fig. 7. Spatial orthogonality of the two signals at the $(-)$ output port requires [with Eqs. (19) and (21)]

$$\frac{|g_{o_1}{}^-|^2}{|g_{o_2}{}^-|^2} = -\frac{\langle o_2 | e_1\rangle\langle e_2 | o_2\rangle}{\langle o_1 | e_1\rangle\langle e_2 | o_1\rangle} = \frac{I_{e_2}}{I_{e_1}}\frac{\sin^2\theta_2}{\cos^2\theta_1}. \tag{22}$$

Using definition (18) for the gain, one can find conditions on the beam-splitter ratio r and the coupling strength Γl that satisfy Eq. (22). One may, for example, fix the beam-splitter ratio and adjust the coupling strength by illuminating the photorefractive medium with a variable intensity incoherent beam. An example of the evolution of the overlaps $h^{\pm}$ is shown in Fig. 8. At a coupling strength of $\Gamma l \approx 7$ the signals are orthogonalized at the $(-)$ port, while the spatial overlap is increased at the $(+)$ port. For large values of Γl the overlap at the $(+)$ port tends asymptotically to the input overlap because all the incident energy is transferred to the $(+)$ port, while the spatial overlap at the $(-)$ port remains at a relatively low level.

When there are more than two signals present it will in general be possible to orthogonalize only one pair of signals with a single reflexive interaction. To orthogonalize a series of $N - 1$ interactions would require N signals. The procedure for doing so is not particularly straightforward. The evolution of the spatial overlap may in general be found by direct numerical integration of Eqs. (4) and (5).

The procedure for analyzing the coupling of spatially overlapping signals may be reversed. We showed that a set of temporally orthogonal but spatially overlapping signals is equivalent to a linear superposition of different signals that are temporally and spatially orthogonal. Going backward implies that temporally and spatially orthogonal signals, such as those studied in Section 3 above, are equivalent to different temporally orthogonal but spatially overlapping signals. These signals may then be spatially orthogonalized by reflexive coupling. The implication is that reflexive coupling allows temporally and spatially orthogonal signals to be transformed into new temporally and spatially orthogonal signals with different temporal and spatial basis functions. In this way quite general transformations of the spatiotemporal nature of an optical beam are possible.

In summary, we have shown that reflexive coupling of a multisignal beam is useful for manipulating the information content of the beam and the individual signal strengths. Depending on the choice of output port and the parameters of the reflexive interaction it is possible to enhance strong signals, extract weak signals out of a strong background, or equalize signals with different intensities. The spatial similarity of different signals may also be modified such that spatially overlapping signals become spatially orthogonal.

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